Answers to chapter 3 review questions

3.1 Explain why the indifference curves in a probability triangle diagram are straight lines if preferences satisfy expected utility theory.

The expected utility of a prospect \((p_1, x_1; p_2, x_2; p_3, x_3)\) is given by

\[
U = p_1 u(x_1) + p_2 u(x_2) + p_3 u(x_3).
\]

We can now use the fact \(p_1 + p_2 + p_3 = 1\) to write \(p_2 = 1 - p_1 - p_3\) and

\[
U = p_1 u(x_1) + (1 - p_1 - p_3) u(x_2) + p_3 u(x_3).
\]

Rearranging a bit we get

\[
U = u(x_2) + p_1 (u(x_1) - u(x_2)) + p_3 (u(x_3) - u(x_2)).
\]

In a probability triangle diagram the values of \(u(x_1), u(x_2)\) and \(u(x_3)\) are fixed. The values of \(p_1\) and \(p_3\) vary. Along an indifference curve the value of \(U\) is constant so we can write

\[
p_3 = \frac{U - u(x_2)}{u(x_3) - u(x_2)} - p_1 \frac{u(x_1) - u(x_2)}{u(x_3) - u(x_2)}.
\]

We can see here that we get a linear relationship between \(p_3\) and \(p_1\). Hence the indifference curves are straight line. Furthermore, they are all parallel because a change in \(U\) just shifts the intercept of the indifference curve.
3.2 Show why it is inconsistent with expected utility that most people choose prospect F over E and prospect G over H, when the prospects are as in Table 3.5.

Prospect F is $(0.9, 3000; 0.1, 0)$ and prospect E is $(0.45, 6000; 0.55, 0)$. If a person prefers prospect F over E and her preferences are consistent with expected utility then

$$0.9u(3000) + 0.1u(0) > 0.45u(6000) + 0.55u(0).$$

Which we can rewrite

$$0.9u(3000) > 0.45u(6000) + 0.45u(0).$$

It is convenient to divide everything by $0.45u(6000)$ to give

$$2 \frac{u(3000)}{u(6000)} > 1 + \frac{u(0)}{u(6000)}.$$

Prospect G is $(0.001, 6000; 0.999, 0)$ and prospect H is $(0.002, 3000; 0.998, 0)$. If a person prefers prospect G over H and her preferences are consistent with expected utility then

$$0.001u(6000) + 0.999u(0) > 0.002u(3000) + 0.998u(0).$$

Which we can rewrite

$$0.001u(6000) + 0.001u(0) > 0.002u(3000).$$

It is convenient to divide everything by $0.001u(6000)$ to give

$$1 + \frac{u(0)}{u(6000)} > 2 \frac{u(3000)}{u(6000)}.$$

But, here we can see the problem. The two inequalities we have derived are contradictory. So, this person’s preferences are not consistent with expected utility maximization.
3.3 Using the model of disappointment with $\theta = 0.002$ consider the following three prospects
$A = (0.5, 2400; 0.5, 0), B = (0.7, 2400; 0.3, 0)$ and $C = (0.3, 2400; 0.7, 0)$. Work out the utility of each prospect and comment on the result.

Let us look at each prospect in turn starting with prospect A. I shall set $u(x) = x$. The expected utility is

$$prior = 0.5u(2400) + 0.5u(0) = 1200.$$ 

So the utility of the prospect is

$$U(A) = 0.5[u(2400) + \theta(u(2400) - prior)^2] + 0.5[u(0) - \theta(u(0) - prior)^2].$$

This simplifies to

$$U(A) = 0.5[2400 + 0.002(2400 - 1200)^2] + 0.5[0 - 0.002(0 - 1200)^2].$$

Which gives $U(A) = 1200$. Note that in this case the expected elation from getting $2400$ is exactly offset by the expected disappointment from getting $0$.

Next we can look at prospect B. The expected utility is

$$prior = 0.7u(2400) + 0.3u(0) = 1680.$$ 

So the utility of the prospect is

$$U(B) = 0.7[u(2400) + \theta(u(2400) - prior)^2] + 0.3[u(0) - \theta(u(0) - prior)^2].$$

This simplifies to

$$U(B) = 0.7[2400 + 0.002(2400 - 1680)^2] + 0.3[0 - 0.002(0 - 1680)^2].$$

Which gives $U(B) = 712$. In this case the disappointment from only getting $0$ is so large that it outweighs the higher expected payoff. The person prefers prospect A.

Finally we can look at prospect C. The expected utility is

$$prior = 0.3u(2400) + 0.7u(0) = 720.$$ 

So the utility of the prospect is

$$U(C) = 0.3[2400 + 0.002(2400 - 720)^2] + 0.7[0 - 0.002(0 - 720)^2].$$

Which gives $U(C) = 1688$. In this case the prior expectation is low and so the elation from getting $2400$ is enough to outweigh the lower expected payoff. Overall, this gives the perverse result that the person prefers prospect C over prospect A over prospect B. This result reflects extreme aversion against disappointment.
3.4 Using prospect theory say whether a person would prefer prospect I or prospects J to N from Table 3.6.

With prospect I the person gains or loses nothing so \( v(0) = 0 \). With prospect J the person gains $100 with probability 0.5 and loses $105 with probability 0.5. The 0.5 probability of a gain is given decision weight

\[
\pi^g(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^\frac{1}{\gamma}} = \frac{0.5^{0.61}}{(0.5^{0.61} + 0.5^{0.61})^{0.61}} = 0.42.
\]

The 0.5 probability of a loss is given decision weight

\[
\pi^l(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^\frac{1}{\delta}} = \frac{0.5^{0.69}}{(0.5^{0.69} + 0.5^{0.69})^{0.69}} = 0.46.
\]

So the expected value from prospect J is

\[
U(J) = 0.42v(105) + 0.46v(-100).
\]

Substituting in the formula from the text we get

\[
U(J) = 0.42 \times 105^{0.88} - 0.46 \times 2.25 \times 100^{0.88} = -34.4.
\]

So the person would prefer I over J.

If we look at prospect K we get

\[
U(K) = 0.42 \times 125^{0.88} - 0.46 \times 2.25 \times 100^{0.88} = -30.1.
\]

So, he still prefers prospect I. For prospect L

\[
U(L) = 0.42 \times 200^{0.88} - 0.46 \times 2.25 \times 100^{0.88} = -15.1.
\]

So, he still prefers prospect I. With prospect M we get

\[
U(M) = 0.42 \times 375^{0.88} - 0.46 \times 2.25 \times 225^{0.88} = -44.2.
\]

So, he still prefers prospect I. The overweighting of the probability of losing coupled with loss aversion can explain the reluctance to gamble. But, there are limits. With prospect N we get

\[
U(N) = 0.42 \times 36000000^{0.88} - 0.46 \times 2.25 \times 600^{0.88} = 1,873,824.
\]

So, this is clearly preferred to prospect I. Prospect theory, therefore, can cope with risk aversion over small gambles without imposing extreme risk aversion over large gambles.
3.5 In 1963 Paul Samuelson wrote about a colleague who said that he would turn down the prospect \((0.5, -100; 0.5, 200)\) but would accept 100 such prospects. Suppose that his utility function is \(u(x) = x - w\) if \(x \geq w\) and \(u(x) = -2.5(w - x)\) if \(w > x\) where \(w\) is his wealth. Show why he turned down the prospect?

His expected utility is

\[
U(\text{gamble}) = 0.5u(w + $200) + 0.5u(w - $100).
\]

This gives us

\[
U(\text{gamble}) = 0.5 \times 200 - 0.5 \times 2.5 \times 100 = -25.
\]

He should not take the gamble.

*Now imagine two prospects will be done in turn and he will adjust his wealth level after each prospect. Show that he should turn down the two prospects?*

Given that he revises his wealth level after each gamble the calculations above still hold. The expected utility of each gamble is \(-25\) and so he should not take them.

*Imagine that he only adjust his wealth level after seeing the outcome of both prospects. Should he take on the two prospects?*

The easiest way to deal with this case is to treat the two prospects as one big combined prospect. With probability \(0.25 = 0.5 \times 0.5\) he wins both gambles and gets $400. With probability \(0.25 = 0.5 \times 0.5\) he loses both gambles and loses $200. With probability \(0.5\) he wins one gamble and loses the other giving a net gain of $100. So, his expected utility is

\[
U(2\text{gambles}) = 0.25u(w + $400) + 0.5u(w + $100) + 0.25u(w - $200).
\]

This gives us

\[
U(2\text{gambles}) = 0.25 \times 400 + 0.5 \times 100 - 0.25 \times 2.5 \times 200 = 25.
\]

He should take the gamble

*Should he take on 100 prospects?*

It is far to tedious to work out the expected utility in this case. But, the answer is yes. With 100 prospects the expected value is \(50 \times 100 = $5000\). More importantly, there is very little chance of him losing money. Specifically, he only needs to win on 34 or more of the gambles to make money overall. The probability of 34 or more wins is 0.9995.
3.6 What do you think is the relevant reference point of a prospect? How might the certainty effect be related to reference dependence?

The reference point may be the expected value or expected utility of the prospect, as in a model of disappointment. The reference point will, however, be highly influenced by expectations. For example, someone who expected to choose the prospect will likely have a very different reference point to someone who did not expect to choose the prospect. The reference point, therefore, will be highly context dependent.

The certainty effect can be explained by reference dependence, but we need to be very careful what the reference point is. To illustrate we can look at how prospect theory can explain the Allais Paradox. To focus the discussion I will ignore the weighting of probability, which is covered in the textbook (see Table 3.11).

Recall that prospect A is $(0.33, $2500; 0.66, $2400; 0.01, $0)$ and prospect B is $(1, $2400)$. The expected value of prospect A is $2409 and that of prospect B is clearly $2400. Suppose we use these as reference points. Then we get

$$U(A) = 0.33v(2500 - 2409) + 0.66v(2400 - 2409) + 0.01v(0 - 2409).$$

Using the standard formulation of the value function this gives

$$U(A) = 0.33(2500 - 2409)^{0.88} - 0.66 \times 2.25 \times (2409 - 2400)^{0.88} - 0.01 \times 2.25(0 - 2409)^{0.88} = -14.$$  

While

$$U(B) = v(2400 - 2400) = 0.$$  

So, the person would prefer prospect B over A. This is what we observe with the Allais paradox.

Recall that prospect C is $(0.33, $2500; 0.67, $0)$ and prospect D is $(0.34, $2400; 0.66, $0)$. The expected value of prospect C is $825 and that of prospect D is clearly $816. Suppose we use these as reference points. Then

$$U(C) = 0.33(2500 - 825)^{0.88} - 0.67 \times 2.25 \times (825 - 0)^{0.88} = -328.$$  

While

$$U(D) = 0.34(2400 - 816)^{0.88} - 0.66 \times 2.25 \times (816 - 0)^{0.88} = -319.$$  

So, the person is predicted to prefer prospect D over C. This, however, is NOT what we observe with the Allais paradox.

So, how can we explain the Allais paradox? An alternative is to use a reference point of $2400 for prospects A and B and a reference point of $0 for prospects C and D. The logic
would be that the sight of prospect B makes the person feel as though they already have $2400, because they only need to choose prospect B and they have it. This is the certainty effect. Prospects C and D, by contrast, do not create a positive reference point because there is no certain gain. In this case

\[ U(A) = 0.33(2500 - 2400)^{0.88} - 0.66 \times 2.25 \times (2400 - 2400)^{0.88} - 0.01 \]
\[ \times 2.25(0 - 2400)^{0.88} = -2 \]

while \( U(B) = 0 \). The person would still prefer prospect B over A. While

\[ U(C) = 0.33(2500 - 0)^{0.88} + 0.67(0 - 0)^{0.88} = 322 \]

and

\[ U(D) = 0.34(2400 - 0)^{0.88} + 0.66(0 - 0)^{0.88} = 321. \]

So, the person is now predicted to prefer C over D. This is consistent with the Allais paradox.

This discussion illustrates how important the reference point can be. It also illustrates potentially framing effects. The certainty effect may be caused by a discontinuous jump in the reference point when someone is faced with a prospect with a certain gain. Prospect B with its certain gain changes the reference point while prospects C and D do not. A different framing, however, may lead people to use a reference point based on expected value. As we have seen this alternative framing would lead to different choice.
3.7 One set of prospects considered by Loomes, Starmer and Sugden (1991) was the following
\( A = (0.4, $10; 0.6, $3) \), \( B = (0.7, $7.50; 0.3, $1) \) and \( C = (1, $5) \). What would you choose between A and B, B and C, and A and C? How can regret theory help us explain choices in this case?

First of all let me say what the subjects in the study did. The most popular choice (chosen by around 32% of subjects) was \( A > C > B \). A close second (chosen by around 28% of subjects) was the intransitive cycle \( B > A \) and \( C > B \) and \( A > C \). Of most interest, therefore, is to show that intransitive preferences like this are permissible with regret theory.

Let us compare prospect A and B. If the person chooses A there is a 0.4 probability he will rejoice \( R(10,7.50) \), a 0.3 probability he will regret \( R(3,7.50) \) and a 0.3 probability he will rejoice \( R(3,1) \). So he will choose prospect A if

\[
0.4R(10,7.50) + 0.3R(3,7.50) + 0.3R(3,1) > 0.
\]

Let us next compare prospect A and C. If the person chooses A there is a 0.4 probability he will rejoice \( R(10,5) \) and a 0.6 probability he will regret \( R(3,5) \). So he will choose prospect A if

\[
0.4R(10,5) + 0.6R(3,5) > 0.
\]

Finally we can compare prospect B and C. If the person chooses B there is a 0.7 probability he will rejoice \( R(7.50,5) \) and a 0.3 probability he will regret \( R(1,5) \). So he will choose prospect B if

\[
0.7R(7.50,5) + 0.3R(1,5) > 0.
\]

Is it possible someone could prefer B to A, C to B and A to C? For this to be possible we need

\[
0.4R(10,7.50) + 0.3R(3,7.50) + 0.3R(3,1) < 0  \\
0.4R(10,5) + 0.6R(3,5) + 0.3R(3,5) > 0  \\
0.4R(7.50,5) + 0.3R(7.50,5) + 0.3R(1,5) < 0.
\]

This implies

\[
0.4[R(10,7.5) + R(7,55) - R(10,5)] + 0.3[R(3,7.5) + R(7.55,5) - R(3,5)]  \\
+ 0.3[R(3,1) + R(1,5) - R(3,5)] < 0.
\]

Note, however, that each of the terms in brackets must be non-negative by assumption. For example, \( R(10,5) > R(10,7.5) + R(7.5,5) \). Thus, cycles of the type observed are possible with regret theory.
3.8 What would happen to the equity premium if investors were less loss averse or the evaluation period was longer? How often should you evaluate your investments?

If we use a prospect theory model to solve the equity premium puzzle then: we would predict the equity premium will fall if investors are less loss averse or use a longer evaluation period. If an investor is less loss averse then the losses she is likely to experience with a risky asset will feel be perceived as less bad. With a longer evaluation period the investor is less likely to experience a loss when investing in the risky asset. In both cases the relative returns on gains do not need to be as high because there are less perceived losses to compensate.

A longer evaluation period means you will not be ‘scared’ by loss aversion. This means you can benefit from an equity premium and get a high return (and high utility) from investing in risky assets. But, obviously it is important to check on your investments and make changes if necessary. The key to good investing is, therefore, to keep check of your investments without evaluating the success of your portfolio. This way you can avoid the psychological cost of losses while still managing your investments. Of course, avoiding the psychological cost of losses may be either said than done. What you may want to avoid, however, are things like monthly and annual statements of account.
3.9 I argued in section 3.7 that, applying the standard formulation of prospect theory, a person would pay $100 for an reduced deductible of $500 if the claim rate was four percent. How much would a person be willing to pay to reduce the deductible by $165 if the claim rate was 25 percent? Comment on the numbers in Table 3.18.

Let us first of all work through the theory for a four percent claim rate. The no loss in buying hypothesis means that paying the $100 for a reduced deductible is not counted as a loss. So, if the person does not buy the reduced deductible she gains $100 but with a four percent chance will lose out because she has to make a claim. Consequently, she has chosen prospect $N = (0.96, $100; 0.04, −$400)$. If the person buys the reduced deductible she does not gain, but cannot lose. So, she has chosen prospect $B = (1,0)$.

If we set $\delta = \gamma = 0.61$ then the 0.04 probability of a loss is given decision weight

$$
\pi^L(p) = \frac{p^\delta}{(p^\delta + (1 - p)^\delta)^\gamma} = \frac{0.04^{0.61}}{(0.04^{0.61} + 0.96^{0.61})^{0.61}} = 0.12.
$$

The 0.96 probability of a gain is given decision weight

$$
\pi^G(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^\delta} = \frac{0.96^{0.61}}{(0.96^{0.61} + 0.04^{0.61})^{0.61}} = 0.82.
$$

So the expected value from prospect $N$ is

$$
U(N) = 0.82v(100) + 0.12v(−$400).
$$

Substituting in the standard formula we get

$$
U(N) = 0.82 \times 100^{0.88} - 0.12 \times 2.25 \times 400^{0.88} = -5.43.
$$

So, the person prefers the reduced deductible. Note that if we use $\delta = 0.69$ she would prefer to not buy the reduced deductible! It is really important, therefore, we know how people weight probabilities.

We can now work through the theory for a 25 percent claim rate. Suppose the person needs to pay an extra $x$ to reduce the deductible by $165. If the person does not buy the reduced deductible she gains $x$ but with a 25 percent chance will lose out because she has to make a claim. Consequently, she has chosen prospect $N = (0.75, x; 0.25, x − 165)$. If the person buys the extra deductible she does not gain, but cannot lose. So, she has chosen prospect $B = (1,0)$.

If we set $\delta = \gamma = 0.61$ then the 0.25 probability of a loss is given decision weight

$$
\pi^L(p) = \frac{0.25^{0.61}}{(0.25^{0.61} + 0.75^{0.61})^{0.61}} = 0.29.
$$
The 0.75 probability of a gain is given decision weight

\[ \pi^d(p) = \frac{0.75^{0.61}}{(0.75^{0.61} + 0.25^{0.61})^{0.61}} = 0.57. \]

So the expected value from prospect N is

\[ U(N) = 0.57v(x) + 0.29v(x - 165). \]

Substituting in the standard formula we get

\[ U(N) = 0.57 \times x^{0.88} - 0.29 \times 2.25 \times (165 - x)^{0.88}. \]

We need to find a value of $x$ such that $U(N) = U(B) = 0$. A value of $x = 88.8$ is near enough. So the person would pay up to $88.8 to reduce the deductible. Given that the extra premium was only around $55 it is no surprise that many opted for the regular rather than low deductible in the Israeli car insurance example (Table 3.18).

3.10 Is expected utility theory of any practical relevance?

Given the very strong evidence of systematic violations from expected utility maximization it would be tempting to think that expected utility theory has no practical relevance. But, that would be too extreme a view. The objective in economics is to have simple models that tell us something useful about behavior. The primary virtue of expected utility theory is its simplicity. Models that account for disappointment, regret, loss aversion, weighting of probabilities and the like may be more accurate but they are also considerably more difficult to apply. And in many instances expected utility theory will be accurate enough to give a useful account of behaviour.

Expected utility theory is, therefore, of practical relevance. It is important, however, we keep in mind its limitations. That way we can better know when a more complex model of behavior is warranted. For example, risky choices with losses and gains and small probabilities are a warning sign that expected utility theory may not be best.