CHAPTER 77 SOLUTION OF FIRST-ORDER DIFFERENTIAL EQUATIONS BY SEPARABLE VARIABLES

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1. Sketch a family of curves represented by each of the following differential equations:

(a) \( \frac{dy}{dx} = 6 \)  
(b) \( \frac{dy}{dx} = 3x \)  
(c) \( \frac{dy}{dx} = x + 2 \)

(a) If \( \frac{dy}{dx} = 6 \), then \( y = \int 6 \, dx = 6x + c \)

There are an infinite number of graphs of \( y = 6x + c \); three curves are shown below.

(b) If \( \frac{dy}{dx} = 3x \), then \( y = \int 3x \, dx = \frac{3}{2} x^2 + c \)

A family of three typical curves is shown below.
(c) If \( \frac{dy}{dx} = x + 2 \), then \( y = \int (x + 2) \, dx = \frac{x^2}{2} + 2x + c \)

A family of three typical curves is shown below.

2. Sketch the family of curves given by the equation \( \frac{dy}{dx} = 2x + 3 \) and determine the equation of one of these curves which passes through the point (1, 3).

If \( \frac{dy}{dx} = 2x + 3 \), then \( y = \int (2x + 3) \, dx = x^2 + 3x + c \)

If the curve passes through the point (1, 3) then \( x = 1 \) and \( y = 3 \)

Hence, \( 3 = (1)^2 + 3(1) + c \) i.e. \( c = -1 \)

and \( y = x^2 + 3x - 1 \)

A family of three curves is shown below, including \( y = x^2 + 3x - 1 \), which passes through the point (1, 3)
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1. Solve: \( \frac{dy}{dx} = \cos 4x - 2x \)

Since \( \frac{dy}{dx} = \cos 4x - 2x \) \quad y = \int (\cos 4x - 2x) \, dx

i.e. \( y = \frac{1}{4} \sin 4x - x^2 + c \)

2. Solve: \( 2x \frac{dy}{dx} = 3 - x^3 \)

Since \( 2x \frac{dy}{dx} = 3 - x^3 \) \quad i.e. \( \frac{dy}{dx} = \frac{3 - x^3}{2x} = \frac{3}{2x} - \frac{x^3}{2} \)

Hence, \( y = \int \left( \frac{3}{2x} - \frac{x^3}{2} \right) \, dx = \frac{3}{2} \ln x - \frac{x^3}{6} + c \)

3. Solve: \( \frac{dy}{dx} + x = 3 \), given \( y = 2 \) when \( x = 1 \)

If \( \frac{dy}{dx} + x = 3 \), then \( \frac{dy}{dx} = 3 - x \) \quad and \( y = \int (3 - x) \, dx = 3x - \frac{x^2}{2} + c \)

If \( y = 2 \) when \( x = 1 \), then \( 2 = 3 - \frac{1}{2} + c \) \quad from which, \( c = -\frac{1}{2} \)

Hence, \( y = 3x - \frac{x^2}{2} - \frac{1}{2} \)

4. Solve: \( 3 \frac{dy}{d\theta} + \sin \theta = 0 \), given \( y = \frac{2}{3} \) when \( \theta = \frac{\pi}{3} \)

Since \( 3 \frac{dy}{d\theta} + \sin \theta = 0 \) \quad i.e. \( \frac{dy}{d\theta} = -\frac{1}{3} \sin \theta \)

and \( y = \int -\frac{1}{3} \sin \theta \, d\theta = \left( -\frac{1}{3} \right)(-\cos \theta) + c = \frac{1}{3} \cos \theta + c \)
\[ y = \frac{2}{3} \text{ when } \theta = \frac{\pi}{3} \text{ hence, } \frac{2}{3} = \frac{1}{3} \cos \frac{\pi}{3} + c \]

i.e. \[ \frac{2}{3} = \frac{1}{3} \left( \frac{1}{2} \right) + c \quad \text{i.e.} \quad \frac{2}{3} - \frac{1}{3} \left( \frac{1}{2} \right) = c \quad \text{i.e.} \quad c = \frac{1}{2} \]

Hence, \[ y = \frac{1}{3} \cos \theta + \frac{1}{2} \]

5. Solve: \( \frac{1}{e^x} + 2 = x - 3 \frac{dy}{dx} \), given \( y = 1 \) when \( x = 0 \)

If \( \frac{1}{e^x} + 2 = x - 3 \frac{dy}{dx} \) then \( 3 \frac{dy}{dx} = x - e^{-x} - 2 \) and \( \frac{dy}{dx} = \frac{1}{3} (x - e^{-x} - 2) \)

Hence, \[ y = \frac{1}{3} \int \left(x - e^{-x} - 2\right) dx = \frac{1}{3} \left( \frac{x^2}{2} + e^{-x} - 2x \right) + c \]

If \( y = 1 \) when \( x = 0 \), then \( 1 = \frac{1}{3} (0 + 1 + 0) + c \quad \text{i.e.} \quad c = \frac{2}{3} \)

Thus, \[ y = \frac{1}{3} \left( \frac{x^2}{2} + e^{-x} - 2x \right) + \frac{2}{3} \quad \text{or} \quad \frac{1}{6} \left( x^2 - 4x + \frac{2}{e^x} + 4 \right) \]

6. The gradient of a curve is given by \( \frac{dy}{dx} + \frac{x^2}{2} = 3x \). Find the equation of the curve if it passes through the point \( (1, \frac{1}{3}) \).

If \( \frac{dy}{dx} + \frac{x^2}{2} = 3x \), then \( \frac{dy}{dx} = 3x - \frac{x^2}{2} \)

Hence, \[ y = \int \left( 3x - \frac{x^2}{2} \right) dx = \frac{3x^2}{2} - \frac{x^3}{6} + c \]

If it passes through \( \left(1, \frac{1}{3}\right) \), \( x = 1 \) and \( y = \frac{1}{3} \)

Thus, \( \frac{1}{3} = \frac{3}{2} - \frac{1}{6} + c \quad \text{from which,} \quad c = \frac{1}{3} - \frac{3}{2} + \frac{1}{6} = -1 \)

Hence, \[ y = \frac{3}{2} x^2 - \frac{x^3}{6} - 1 \]
7. The acceleration \( a \) of a body is equal to its rate of change of velocity, \( \frac{dv}{dt} \). Find an equation for \( v \) in terms of \( t \), given that when \( t = 0 \), velocity \( v = u \).

\[
\frac{dv}{dt} = a \quad \text{hence,} \quad v = \int a \, dt = at + c
\]

When \( t = 0 \), velocity \( v = u \), hence, \( u = 0 + c \) from which, \( c = u \)

Hence, velocity, \( v = at + u \) or \( v = u + at \)

8. An object is thrown vertically upwards with an initial velocity \( u \) of 20 m/s. The motion of the object follows the differential equation \( \frac{ds}{dt} = u - gt \), where \( s \) is the height of the object in metres at time \( t \) seconds and \( g = 9.8 \text{ m/s}^2 \). Determine the height of the object after 3 seconds if \( s = 0 \) when \( t = 0 \)

If \( \frac{ds}{dt} = u - gt \), then \( s = \int (u - gt) \, dt = ut - \frac{gt^2}{2} + c \)

Since \( s = 0 \) when \( t = 0 \), then \( c = 0 \)

Hence, \( s = ut - \frac{gt^2}{2} \) and if \( u = 20 \) and \( s = 9.8 \), then \( s = 20t - \frac{9.8t^2}{2} \) i.e. \( u = 20t - 4.9t^2 \)

The height when \( t = 3 \), \( s = 3(20) - 4.9(3)^2 \)

i.e. height = 60 - 44.1 = 15.9 m
1. Solve: \( \frac{dy}{dx} = 2 + 3y \)

Rearranging \( \frac{dy}{dx} = 2 + 3y \) gives: \( dx = \frac{dy}{2+3y} \)

Integrating both sides gives:
\[
\int dx = \int \frac{dy}{2+3y}
\]

Thus, by using the substitution \( u = (2 + 3y) \),
\[
x = \frac{1}{3} \ln(2 + 3y) + c
\]

2. Solve: \( \frac{dy}{dx} = 2 \cos^2 y \)

If \( \frac{dy}{dx} = 2 \cos^2 y \) then \( \frac{dy}{\cos^2 y} = 2 \, dx \)

and
\[
\int \sec^2 y \, dy = \int 2 \, dx
\]

i.e.
\[
\tan y = 2x + c
\]

3. Solve: \( (y^2 + 2) \frac{dy}{dx} = 5y \), given \( y = 1 \) when \( x = \frac{1}{2} \)

If \( (y^2 + 2) \frac{dy}{dx} = 5y \) then \( \left( \frac{y^2 + 2}{y} \right) dy = 5 \, dx \)

and
\[
\int \left( \frac{y^2 + 2}{y} \right) \, dy = \int 5 \, dx
\]

i.e.
\[
\frac{y^2}{2} + 2 \ln y = 5x + c
\]

\( y = 1 \) when \( x = \frac{1}{2} \), hence,
\[
\frac{1}{2} + 2 \ln 1 = \frac{5}{2} + c \quad \text{from which,} \quad c = \frac{1}{2} - \frac{5}{2} = -2
\]

and
\[
\frac{y^2}{2} + 2 \ln y = 5x - 2
\]
4. The current in an electric circuit is given by the equation \( Ri + L \frac{di}{dt} = 0 \), where \( L \) and \( R \) are constants.

Show that \( i = I e^{-\frac{Ri}{L}} \), given that \( i = I \) when \( t = 0 \)

If \( Ri + L \frac{di}{dt} = 0 \), then \( L \frac{di}{dt} = -Ri \)

and \( \frac{di}{dt} = -\frac{Ri}{L} \)

from which, \( \frac{di}{i} = -\frac{R}{L} \, dt \) and \( \int \frac{di}{i} = \int -\frac{R}{L} \, dt \)

Thus, \( \ln i = -\frac{Rt}{L} + c \)

\( i = I \) when \( t = 0 \), thus \( \ln I = c \)

Hence, \( \ln i = -\frac{Rt}{L} + \ln I \)

i.e. \( \ln i - \ln I = -\frac{Rt}{L} \)

i.e. \( \ln \left(\frac{i}{I}\right) = -\frac{Rt}{L} \)

Taking anti-logarithms gives: \( \frac{i}{I} = e^{-\frac{Rt}{L}} \) and \( i = I e^{-\frac{Rt}{L}} \)

5. The velocity of a chemical reaction is given by \( \frac{dx}{dt} = k(a - x) \), where \( x \) is the amount transferred in time \( t \), \( k \) is a constant and \( a \) is the concentration at time \( t = 0 \) when \( x = 0 \). Solve the equation and determine \( x \) in terms of \( t \).

If \( \frac{dx}{dt} = k(a - x) \), then \( \frac{dx}{a - x} = k \, dt \)

and \( \int \frac{dx}{a - x} = \int k \, dt \)

i.e. \( -\ln(a - x) = kt + c \)

\( t = 0 \) when \( x = 0 \), hence \( -\ln a = c \)

Thus, \( -\ln(a - x) = kt - \ln a \)
\[ \ln a - \ln(a - x) = kt \]

i.e.

\[ \ln \left( \frac{a}{a - x} \right) = kt \]

and

\[ \frac{a}{a - x} = e^{kt} \]

i.e.

\[ \frac{a}{e^{kt}} = a - x \quad \text{i.e.} \quad a e^{-kt} = a - x \]

and

\[ x = a - a e^{-kt} \quad \text{i.e.} \quad x = a(1 - e^{-kt}) \]

6. (a) Charge \( Q \) coulombs at time \( t \) seconds is given by the differential equation \( R \frac{dQ}{dt} + \frac{Q}{C} = 0 \), where 

\( C \) is the capacitance in farads and \( R \) the resistance in ohms. Solve the equation for \( Q \) given that 

\( Q = Q_0 \) when \( t = 0 \)

(b) A circuit possesses a resistance of \( 250 \times 10^3 \) ohms and a capacitance of \( 8.5 \times 10^{-6} \) farads, and after 

0.32 seconds the charge falls to 8.0 C. Determine the initial charge and the charge after 1 second, 

each correct to 3 significant figures.

(a) If \( R \frac{dQ}{dt} + \frac{Q}{C} = 0 \) then 

\[ \frac{dQ}{dt} = -\frac{Q}{RC} \]

i.e.

\[ \int \frac{dQ}{Q} = \int -\frac{1}{RC} \, dt \]

i.e.

\[ \ln Q = -\frac{t}{RC} + k \]

\( Q = Q_0 \) when \( t = 0 \), hence, \( \ln Q_0 = k \)

Hence,

\[ \ln Q = -\frac{t}{RC} + \ln Q_0 \]

i.e.

\[ \ln Q - \ln Q_0 = -\frac{t}{RC} \]

i.e.

\[ \ln \frac{Q}{Q_0} = -\frac{t}{RC} \]

and

\[ \frac{Q}{Q_0} = e^{-\frac{t}{RC}} \quad \text{and} \quad Q = Q_0 e^{-\frac{t}{RC}} \]

(b) \( R = 250 \times 10^3 \Omega \), \( C = 8.5 \times 10^{-6} \) F, \( t = 0.32 \) s and \( Q = 8.0 \) C
Hence, \[ 8.0 = Q_0 e^{-0.32}, \quad 8.0 = Q_0 (0.8602) \]
from which, initial charge, \( Q_0 = \frac{8.0}{0.8602} = 9.30 \) C

When \( t = 1 \) s, charge, \( Q = Q_0 e^{-\frac{t}{CR}} = 9.30 e^{-\frac{1}{8.5 \times 10^{-5} \times 250 \times 10^5}} = 5.81 \) C

7. A differential equation relating the difference in tension \( T \), pulley contact angle \( \theta \) and coefficient of friction \( \mu \) is \( \frac{dT}{d\theta} = \mu T \). When \( \theta = 0, T = 150 \) N, and \( \mu = 0.30 \) as slipping starts. Determine the tension at the point of slipping when \( \theta = 2 \) radians. Determine also the value of \( \theta \) when \( T \) is 300 N.

Since \( \frac{dT}{d\theta} = \mu T \) then \( \frac{dT}{T} = \mu d\theta \)

and \[ \int \frac{dT}{T} = \int \mu d\theta \]
i.e. \[ \ln T = \mu \theta + c \]

When \( \theta = 0, T = 150 \) N, and \( \mu = 0.30 \), hence \( \ln 150 = (0.30)(0) + c \)
from which, \( c = \ln 150 \)

Hence, \( \ln T = \mu \theta + \ln 150 \)
i.e. \( \ln T - \ln 150 = \mu \theta \)
i.e. \( \ln \left( \frac{T}{150} \right) = \mu \theta \)

from which, \( \frac{T}{150} = e^{\mu \theta} \) and \( T = 150 e^{\mu \theta} \)

When \( \theta = 2 \) radians, tension, \( T = 150 e^{(0.30)(2)} = 150 e^{0.60} = 273.3 \) N

When \( T = 300 \) N, \( 300 = 150 e^{(0.30)\theta} \) i.e. \( \frac{300}{150} = e^{(0.30)\theta} \) i.e. \( 2 = e^{(0.30)\theta} \)

Hence, \( \ln 2 = \ln[e^{(0.30)\theta}] = 0.30 \theta \)
from which, contact angle, \( \theta = \frac{1}{0.30} \ln 2 = 2.31 \) rad
8. The rate of cooling of a body is given by \( \frac{d\theta}{dt} = k \theta \), where \( k \) is a constant. If \( \theta = 60^\circ\text{C} \) when \( t = 2 \) minutes and \( \theta = 50^\circ\text{C} \) when \( t = 5 \) minutes, determine the time taken for \( \theta \) to fall to \( 40^\circ\text{C} \), correct to the nearest second.

If \( \frac{d\theta}{dt} = k \theta \) then \( \frac{d\theta}{\theta} = k \, dt \) and \( \int \frac{d\theta}{\theta} = \int k \, dt \)

\[
\text{i.e.} \quad \ln \theta = kt + c
\]

When \( \theta = 60^\circ\text{C}, t = 2, \) i.e. \( \ln 60 = 2k + c \) \hspace{1cm} (1)

When \( \theta = 50^\circ\text{C}, t = 5, \) i.e. \( \ln 50 = 5k + c \) \hspace{1cm} (2)

(1) – (2) gives: \( \ln 60 - \ln 50 = -3k \)

from which, \( k = \frac{-1}{3} \ln \frac{60}{50} = -0.06077 \)

Substituting in (1): \( \ln 60 = 2(-0.06077) + c \)

from which, \( c = \ln 60 + 2(0.06077) = 4.2159 \)

Hence, \( \ln \theta = kt + c = -0.06077t + 4.2159 \)

When \( \theta = 40^\circ\text{C}, \) \( \ln 40 = -0.06077t + 4.2159 \)

and \( \text{time, } t = \frac{4.2159 - \ln 40}{0.06077} = 8.672 \text{ min} = 8 \text{ min} 40 \text{ s} \)
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1. Solve: \( \frac{dy}{dx} = 2y \cos x \)

Since \( \frac{dy}{dx} = 2y \cos x \) then \( \frac{dy}{y} = 2 \cos x \, dx \)

and \( \int \frac{dy}{y} = \int 2 \cos x \, dx \)

i.e. \( \ln y = 2 \sin x + c \)

2. Solve: \( (2y - 1) \frac{dy}{dx} = (3x^2 + 1) \), given \( x = 1 \) when \( y = 2 \)

If \( (2y - 1) \frac{dy}{dx} = (3x^2 + 1) \), then \( \int (2y - 1) \, dy = \int (3x^2 + 1) \, dx \)

i.e. \( y^2 - y = x^3 + x + c \)

\( x = 1 \) when \( y = 2 \), hence, \( 4 - 2 = 1 + 1 + c \) from which, \( c = 0 \)

Thus, \( y^2 - y = x^3 + x \)

3. Solve: \( \frac{dy}{dx} = e^{2x-y} \), given \( x = 0 \) when \( y = 0 \)

\( \frac{dy}{dx} = e^{2x-y} = (e^2)(e^{-y}) \), by the laws of indices

Separating the variables gives: \( \frac{dy}{e^{-y}} = e^{2x} \, dx \) i.e. \( e^y \, dy = e^{2x} \, dx \)

Integrating both sides gives: \( \int e^y \, dy = \int e^{2x} \, dx \)

Thus the general solution is: \( e^y = \frac{1}{2} e^{2x} + c \)

When \( x = 0 \), \( y = 0 \), thus: \( e^0 = \frac{1}{2} e^0 + c \)
from which, \[ c = 1 - \frac{1}{2} = \frac{1}{2} \]

Hence the particular solution is: \[ e^y = \frac{1}{2} e^{2x} + \frac{1}{2} \]

4. Solve: \[ 2y(1 - x) + x(1 + y) \frac{dy}{dx} = 0, \text{ given } x = 1 \text{ when } y = 1 \]

If \( 2y(1 - x) + x(1 + y) \frac{dy}{dx} = 0 \) then \( x(1 + y) \frac{dy}{dx} = -2y(1 - x) = 2y(x - 1) \)

Thus, \[ \int \left( \frac{1+y}{y} \right) dy = \int \left( \frac{2(x-1)}{x} \right) dx \]

i.e. \[ \int \left( \frac{1}{y} + 1 \right) dy = \int \left( 2 - \frac{2}{x} \right) dx \]

\[ \ln y + y = 2x - 2 \ln x + c \]

\( x = 1 \text{ when } y = 1, \text{ hence, } \ln 1 + 1 = 2 - 2 \ln 1 + c \text{ from which, } c = -1 \)

Thus, \[ \ln y + y = 2x - 2 \ln x - 1 \]

or \[ \ln y + 2 \ln x = 2x - y - 1 \]

i.e. \[ \ln y + \ln x^2 = 2x - y - 1 \]

and \[ \ln(x^2y) = 2x - y - 1 \]

5. Show that the solution of the equation \[ \frac{y^2 + 1}{x^2 + 1} = \frac{y}{x} \frac{dy}{dx} \] is of the form \[ \sqrt{\frac{y^2 + 1}{x^2 + 1}} = \text{constant} \]

Since \( \frac{y^2 + 1}{x^2 + 1} = \frac{y}{x} \frac{dy}{dx} \) then \[ \int \frac{y}{y^2 + 1} \frac{dy}{x^2 + 1} = \int \frac{x}{y^2 + 1} \frac{dx}{x^2 + 1} \]

i.e. \[ \frac{1}{2} \ln(y^2 + 1) = \frac{1}{2} \ln(x^2 + 1) + c \]

i.e. \[ \ln(y^2 + 1)^{\frac{1}{2}} - \ln(x^2 + 1)^{\frac{1}{2}} = c \]

i.e. \[ \ln \left( \frac{y^2 + 1}{x^2 + 1} \right)^{\frac{1}{2}} = c \]
or \[ \ln \left( \frac{y^2 + 1}{x^2 + 1} \right) = c \]

and \[ \sqrt{\frac{y^2 + 1}{x^2 + 1}} = e^c = \text{a constant} \]

6. Solve \( xy = (1 - x^2) \frac{dy}{dx} \) for \( y \), given \( x = 0 \) when \( y = 1 \)

Since \( xy = (1 - x^2) \frac{dy}{dx} \) then \[ \frac{xy}{(1 - x^2)} = \frac{dy}{dx} \]

and \[ \frac{dy}{y} = \frac{x}{(1 - x^2)} \, dx \]

i.e. \[ \int \frac{dy}{y} = \int \left( \frac{x}{1 - x^2} \right) \, dx \]

For \( \int \left( \frac{x}{1 - x^2} \right) \, dx \) let \( u = (1 - x^2) \) from which, \[ \frac{du}{dx} = -2x \quad \text{and} \quad dx = \frac{du}{-2x} \]

Hence, \[ \int \left( \frac{x}{1 - x^2} \right) \, dx = \int \frac{x}{u} \, \frac{du}{-2x} = -\frac{1}{2} \int \frac{1}{u} \, du = -\frac{1}{2} \ln u = -\frac{1}{2} \ln (1 - x^2) \]

i.e. if \[ \int \frac{dy}{y} = \int \left( \frac{x}{1 - x^2} \right) \, dx \]

then \[ \ln y = -\frac{1}{2} \ln(1 - x^2) + c \]

\( x = 0 \) when \( y = 1 \), hence, \[ \ln 1 = -\frac{1}{2} \ln 1 + c \quad \text{from which,} \quad c = 0 \]

Hence the particular solution is: \( \ln y = -\frac{1}{2} \ln(1 - x^2) \)

and \[ \ln y = \ln \left( 1 - x^2 \right)^{-\frac{1}{2}} \]

i.e. \[ y = \left( 1 - x^2 \right)^{-\frac{1}{2}} = \frac{1}{\left( 1 - x^2 \right)^{\frac{1}{2}}} \]

i.e. \[ y = \frac{1}{\sqrt{(1 - x^2)}} \]
7. Determine the equation of the curve which satisfies the equation $xy \frac{dy}{dx} = x^2 - 1$, and which passes through the point (1, 2).

Since $xy \frac{dy}{dx} = x^2 - 1$ then $\int y \, dy = \int \frac{x^2 - 1}{x} \, dx = \int \left( x - \frac{1}{x} \right) \, dx$

i.e. $\frac{y^2}{2} = \frac{x^2}{2} - \ln x + c$

If the curve passes through (1, 2) then $x = 1$ and $y = 2$

hence, $\frac{2^2}{2} = \frac{1^2}{2} - \ln 1 + c$ from which, $c = \frac{3}{2}$

Thus, $\frac{y^2}{2} = \frac{x^2}{2} - \ln x + \frac{3}{2}$

or $y^2 = x^2 - 2 \ln x + 3$

8. The p.d., $V$, between the plates of a capacitor $C$ charged by a steady voltage $E$ through a resistor $R$ is given by the equation $CR \frac{dV}{dt} + V = E$

(a) Solve the equation for $V$ given that at $t = 0$, $V = 0$

(b) Calculate $V$, correct to 3 significant figures, when $E = 25$ volts, $C = 20 \times 10^{-6}$ farads, $R = 200 \times 10^3$ ohms and $t = 3.0$ seconds.

(a) Since $CR \frac{dV}{dt} + V = E$ then $\frac{dV}{dt} = \frac{E - V}{CR}$

i.e. $\int \frac{dV}{E - V} = \int \frac{dt}{CR}$

from which, $-\ln (E - V) = \frac{t}{CR} + k$

At $t = 0$, $V = 0$, hence, $-\ln E = k$

Thus, $-\ln (E - V) = \frac{t}{CR} - \ln E$

$\ln E - \ln (E - V) = \frac{t}{CR}$
and 
\[ \ln \left( \frac{E}{E-V} \right) = \frac{t}{CR} \]
i.e. 
\[ \frac{E}{E-V} = e^{\frac{t}{CR}} \]
i.e. 
\[ \frac{E}{e^{\frac{t}{CR}}} = E-V \]
and 
\[ V = E - \frac{E}{e^{\frac{t}{CR}}} = E - E e^{-\frac{t}{CR}} \]
i.e. 
\[ V = E \left( 1 - e^{-\frac{t}{CR}} \right) \text{ volts} \]

(b) Voltage, 
\[ V = E \left( 1 - e^{-\frac{t}{CR}} \right) = 25 \left( 1 - e^{-\frac{3.0}{200x10^{-5}+200x10^{-4}}} \right) = 25 \left( 1 - e^{-0.75} \right) = 13.2 \text{ V} \]

9. Determine the value of \( p \), given that 
\[ x^3 \frac{dy}{dx} = p - x, \] and that \( y = 0 \) when \( x = 2 \) and when \( x = 6 \)

Since \( x^3 \frac{dy}{dx} = p - x \) then 
\[ \int dy = \int \frac{p-x}{x^3} \, dx = \int \left( \frac{p}{x^3} - \frac{1}{x^2} \right) \, dx \]
i.e. 
\[ \int dy = \int \left( px^{-3} - x^{-2} \right) \, dx \]
i.e. 
\[ y = \frac{px^{-2}}{-2} - \frac{x^{-1}}{-1} + c \]
i.e. 
\[ y = -\frac{p}{2x^2} + \frac{1}{x} + c \]
y = 0 when \( x = 2 \), hence, 
\[ 0 = -\frac{p}{8} + \frac{1}{2} + c \quad (1) \]
y = 0 when \( x = 6 \), hence, 
\[ 0 = -\frac{p}{72} + \frac{1}{6} + c \quad (2) \]
(1) – (2) gives: 
\[ 0 = -p \left( \frac{1}{8} - \frac{1}{72} \right) + \left( \frac{1}{2} - \frac{1}{6} \right) \]
i.e. 
\[ 0 = -\frac{p}{9} + \frac{1}{3} \]
i.e. 
\[ \frac{p}{9} = \frac{1}{3} \] from which, \( p = 3 \)