CHAPTER 50 METHODS OF ADDING ALTERNATING WAVEFORMS

EXERCISE 211 Page 577

1. Plot the graph of \( y = 2 \sin A \) from \( A = 0^\circ \) to \( A = 360^\circ \). On the same axes plot \( y = 4 \cos A \). By adding ordinates at intervals plot \( y = 2 \sin A + 4 \cos A \) and obtain a sinusoidal expression for the waveform.

Graphs of \( y = 2 \sin A, \ y = 4 \cos A \) and \( y = 2 \sin A + 4 \cos A \) are shown below

From the graph, \( y = 2 \sin A + 4 \cos A = 4.5 \sin(A + 63.5^\circ) \)
2. Two alternating voltages are given by \( v_1 = 10 \sin \omega t \) volts and \( v_2 = 14 \sin(\omega t + \pi/3) \) volts. By plotting \( v_1 \) and \( v_2 \) on the same axes over one cycle, obtain a sinusoidal expression for (a) \( v_1 + v_2 \) (b) \( v_1 - v_2 \)

(a) \( v_1 = 10 \sin \omega t \), \( v_2 = 14 \sin \left( \omega t + \frac{\pi}{3} \right) \) volts and \( v_1 + v_2 \) are shown sketched below:

\[ v_1 + v_2 \text{ leads } v_1 \text{ by } 36^\circ = 36 \times \frac{\pi}{180} = 0.63 \text{ rad} \]

Hence, by measurement, \( v_1 + v_2 = 20.9 \sin(\omega t + 0.63) \) volts
(b) \( v_1 = 10 \sin \omega t \), \( v_2 = 14 \sin \left( \omega t + \frac{\pi}{3} \right) \) volts and \( v_1 - v_2 \) are shown sketched below:

\[ v_1 - v_2 \text{ lags } v_1 \text{ by } 78^\circ = 78 \times \frac{\pi}{180} = 1.36 \text{ rad} \]

Hence, by measurement, \( v_1 - v_2 = 12.5 \sin (\omega t - 1.36) \) volts

3. Express \( 12 \sin \omega t + 5 \cos \omega t \) in the form \( A \sin(\omega t \pm \alpha) \) by drawing and measurement.

Graphs of \( y = 12 \sin \omega t \), \( y = 5 \cos \omega t \) and \( y = 12 \sin \omega t + 5 \cos \omega t \) are shown below
\( y = 12 \sin \omega t + 5 \cos \omega t \) has a maximum value of 13 and leads \( y = 12 \sin \omega t \) by 22.5°

i.e. \( 22.5 \times \frac{\pi}{180} = 0.393 \) radians

Hence, \( y = 12 \sin \omega t + 5 \cos \omega t = 13 \sin (\omega t + 0.393) \)
EXERCISE 212 Page 579

1. Determine a sinusoidal expression for \(2 \sin \theta + 4 \cos \theta\) by drawing phasors.

The relative positions of \(2 \sin \theta\) and \(4 \cos \theta\) are shown as phasors in diagram (a)

The phasor diagram in diagram (b) is drawn to scale with a ruler and protractor

![Diagram A](image1)

![Diagram B](image2)

The resultant \(R\) is shown and is measured as 4.5 and angle \(\phi\) as 63.5°

Hence, by drawing and measuring: \(2 \sin \theta + 4 \cos \theta = 4.5 \sin(\theta + 63.5°)\)

2. If \(v_1 = 10 \sin \omega t\) volts and \(v_2 = 14 \sin(\omega t + \pi/3)\) volts, determine by drawing phasor sinusoidal expressions for (a) \(v_1 + v_2\) (b) \(v_1 - v_2\)

(a) The relative positions of \(v_1\) and \(v_2\) at time \(t = 0\) are shown as phasors in diagram (a), where \(\frac{\pi}{3}\) rad = 60°

The phasor diagram in diagram (b) is drawn to scale with a ruler and protractor

![Diagram A](image3)

![Diagram B](image4)

The resultant \(v_R\) is shown and is measured as 20.9 V

and angle \(\phi\) as 35.5° or \(\frac{35.5 \times \pi}{180} = 0.62\) rad leading \(v_1\).

Hence, by drawing and measuring: \(v_R = v_1 + v_2 = 20.9 \sin(\omega t + 0.62)\) V
(b) At time $t = 0$, voltage $v_1$ is drawn 10 units long horizontally as shown by $0a$ in the diagram below. Voltage $v_2$ is shown, drawn 14 units long in a broken line and leading by $60^\circ$. The current $-v_2$ is drawn in the opposite direction to the broken line of $v_2$, shown as $ab$ in the diagram. The resultant $v_R$ is given by $0b$ lagging by angle $\phi$.

By measurement, $v_R = 12.5\text{ V}$ and $\phi = 76^\circ$ or $1.33\text{ rad}$.

Hence, by drawing phasors: $v_R = v_1 + v_2 = 12.5 \sin(\omega t - 1.33)\text{ V}$

3. Express $12 \sin \omega t + 5 \cos \omega t$ in the form $A \sin(\omega t \pm \alpha)$ by drawing phasors.

The relative positions of the two phasors at time $t = 0$ are shown in diagram (a).

The phasor diagram in diagram (b) is drawn to scale with a ruler and protractor.

The resultant $R$ is shown and is measured as 13 and angle $\alpha$ as $23^\circ$ or $23\times\frac{\pi}{180} = 0.40\text{ rad}$.

Hence, by drawing and measuring: $12 \sin \omega t + 5 \cos \omega t = 13 \sin(\omega t + 0.40)$
1. Determine, using the cosine and sine rules, a sinusoidal expression for: \( y = 2 \sin A + 4 \cos A \).

The space diagram is shown in (a) below and the phasor diagram is shown in (b)

Using the cosine rule:

\[
R^2 = 2^2 + 4^2 - 2(2)(4) \cos 90^\circ = 20
\]

from which,

\[
R = \sqrt{20} = 4.472
\]

Using the sine rule:

\[
\frac{4}{\sin \theta} = \frac{4.472}{\sin 90^\circ}
\]

from which,

\[
\sin \theta = \frac{4 \sin 90^\circ}{4.472} = 0.894454...
\]

and

\[
\theta = \sin^{-1} 0.894454... = 63.44^\circ
\]

Hence, in sinusoidal form, resultant = 4.472 \sin(\theta + 63.44^\circ)

2. Given \( v_1 = 10 \sin \omega t \) volts and \( v_2 = 14 \sin(\omega t + \pi/3) \) volts, use the cosine and sine rules to determine sinusoidal expressions for (a) \( v_1 + v_2 \) (b) \( v_1 - v_2 \)

(a) The space diagram is shown in (a) below and the phasor diagram is shown in (b)

Using the cosine rule:

\[
v_r^2 = 10^2 + 14^2 - 2(10)(14) \cos 120^\circ = 436
\]

from which,

\[
v_r = \sqrt{436} = 20.88
\]
Using the sine rule:
\[
\frac{14}{\sin \theta} = \frac{20.88}{\sin 120^\circ}
\]
from which, \[
\sin \theta = \frac{14 \sin 120^\circ}{20.88} = 0.580668...
\]
and \[
\theta = \sin^{-1} 0.580668... = 35.50^\circ \text{ or } 0.62 \text{ rad}
\]

Hence, in sinusoidal form, \( v_1 + v_2 = 20.88 \sin(\omega t + 0.62) \) V

(b) \( v_1 - v_2 \) is given by length 0b in the diagram below.

Using the cosine rule:
\[
R^2 = 10^2 + 14^2 - 2(10)(14) \cos 60^\circ = 156
\]
from which, \[
R = \sqrt{156} = 12.50
\]

Using the sine rule:
\[
\frac{14}{\sin \theta} = \frac{12.50}{\sin 60^\circ}
\]
from which, \[
\sin \theta = \frac{14 \sin 60^\circ}{12.50} = 0.969948...
\]
and \[
\theta = \sin^{-1} 0.969948... = 75.92^\circ \text{ or } 1.33 \text{ rad}
\]

Hence, in sinusoidal form, \( v_1 - v_2 = 12.50 \sin(\omega t - 1.33) \) V

3. Express \( 12 \sin \omega t + 5 \cos \omega t \) in the form \( A \sin(\omega t \pm \alpha) \) by using the cosine and sine rules.

The relative positions of the two phasors at time \( t = 0 \) are shown in diagram (a)

The phasor diagram is shown in diagram (b)

Using the cosine rule:
\[
R^2 = 12^2 + 5^2 - 2(12)(5) \cos 90^\circ = 169
\]
from which, \[
R = \sqrt{169} = 13
\]
Using the sine rule: \[
\frac{5}{\sin \phi} = \frac{13}{\sin 90^\circ} \quad \text{from which,} \quad \sin \phi = \frac{5 \sin 90^\circ}{13} = 0.384615...
\]
and \[
\theta = \sin^{-1} 0.384615... = 22.62^\circ \quad \text{or} \quad 0.395 \text{ rad}
\]
Hence, in sinusoidal form, resultant = \[13 \sin(\omega t + 0.395)\]

4. Express \[7 \sin \omega t + 5 \sin \left(\omega t + \frac{\pi}{4}\right)\] in the form \[A \sin(\omega t \pm \alpha)\] by using the cosine and sine rules.

The space diagram is shown in (a) below and the phasor diagram is shown in (b)

Using the cosine rule: \[R^2 = 7^2 + 5^2 - 2(7)(5) \cos 135^\circ = 123.497\]
from which, \[R = \sqrt{123.497} = 11.11\]
Using the sine rule: \[
\frac{5}{\sin \theta} = \frac{11.11}{\sin 135^\circ} \quad \text{from which,} \quad \sin \theta = \frac{5 \sin 135^\circ}{11.11} = 0.31823
\]
and \[
\theta = \sin^{-1} 0.31823 = 18.56^\circ \quad \text{or} \quad 0.324 \text{ rad}
\]
Hence, in sinusoidal form, \[7 \sin \omega t + 5 \sin \left(\omega t + \frac{\pi}{4}\right) = 11.11 \sin(\omega t + 0.324)\]

5. Express \[6 \sin \omega t + 3 \sin \left(\omega t - \frac{\pi}{6}\right)\] in the form \[A \sin(\omega t \pm \alpha)\] by using the cosine and sine rules.

The space diagram is shown in (a) below and the phasor diagram is shown in (b)

Using the cosine rule: \[R^2 = 6^2 + 3^2 - 2(6)(3) \cos 150^\circ = 76.177\]
from which, \[R = \sqrt{76.177} = 8.73\]
Using the sine rule: \[
\frac{3}{\sin \theta} = \frac{8.73}{\sin 150^\circ} \quad \text{from which,} \quad \sin \theta = \frac{3 \sin 150^\circ}{8.73} = 0.171821...
\]
and \[ \theta = \sin^{-1} 0.171821... = 9.89^\circ \text{ or } 0.173 \text{ rad} \]

Hence, in sinusoidal form, \( 6 \sin \omega t + 3 \sin \left( \omega t - \frac{\pi}{6} \right) = 8.73 \sin (\omega t - 0.173) \)

6. The sinusoidal currents in two parallel branches of an electrical network are \( 400 \sin \omega t \) and \( 750 \sin (\omega t - \pi/3) \), both measured in milliamperes. Determine the total current flowing into the parallel arrangement. Give the answer in sinusoidal form and in amperes.

Total current, \( i = 400 \sin \omega t + 750 \sin (\omega t - \pi/3) \) mA

The space diagram is shown in (a) below and the phasor diagram is shown in (b)

Using the cosine rule:
\[
R^2 = 400^2 + 750^2 - 2(400)(750) \cos 120^\circ = 1022 \, 500
\]
from which, \[ R = \sqrt{1022 \, 500} = 1011 \text{ mA} = 1.011 \text{ A} \]

Using the sine rule:
\[
\frac{750}{\sin \phi} = \frac{1011}{\sin 120^\circ}
\]
from which, \[ \sin \phi = \frac{750 \sin 120^\circ}{1011} = 0.642452...
\]

and \[ \theta = \sin^{-1} 0.642452... = 39.97^\circ \text{ or } 0.698 \text{ rad} \]

Hence, in sinusoidal form, \( 400 \sin \omega t + 750 \sin (\omega t - \pi/3) = 1.01 \sin (\omega t - 0.698) \) A

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EXERCISE 214 Page 582

1. Express \( 7 \sin \omega t + 5 \sin \left( \omega t + \frac{\pi}{4} \right) \) in the form \( A \sin(\omega t \pm \alpha) \) by horizontal and vertical components.

From the phasors shown:

Total horizontal component, \( H = 7 \cos 0^\circ + 5 \cos 45^\circ = 10.536 \) \quad (since \( \frac{\pi}{4} \) rad = 45°)

Total vertical component, \( V = 7 \sin 0^\circ + 5 \sin 45^\circ = 3.536 \)

By Pythagoras, the resultant, \( i_r = \sqrt{10.536^2 + 3.536^2} = 11.11 \) A

Phase angle, \( \phi = \tan^{-1} \left( \frac{3.536}{10.536} \right) = 18.55^\circ \) or \( 0.324 \) rad

Hence, by using horizontal and vertical components,

\[
7 \sin \omega t + 5 \sin \left( \omega t + \frac{\pi}{4} \right) = 11.11 \sin (\omega t + 0.324)
\]

2. Express \( 6 \sin \omega t + 3 \sin \left( \omega t - \frac{\pi}{6} \right) \) in the form \( A \sin(\omega t \pm \alpha) \) by horizontal and vertical components.

From the phasors shown:

Total horizontal component, \( H = 6 \cos 0^\circ + 3 \cos(-30^\circ) = 8.598 \) \quad (since \( \frac{\pi}{6} \) rad = 30°)

Total vertical component, \( V = 6 \sin 0^\circ + 3 \sin(-30^\circ) = -1.5 \)

By Pythagoras, the resultant, \( i_r = \sqrt{8.598^2 + 1.5^2} = 8.73 \)
Phase angle, \( \phi = \tan^{-1} \left( \frac{1.5}{8.598} \right) = 9.896^\circ \) or 0.173 rad

Hence, by using horizontal and vertical components,
\[
6 \sin \omega t + 3 \sin \left( \omega t - \frac{\pi}{6} \right) = 8.73 \sin (\omega t - 0.173)
\]

3. Express \( i = 25 \sin \omega t - 15 \sin \left( \omega t + \frac{\pi}{3} \right) \) in the form \( A \sin(\omega t \pm \alpha) \) by horizontal and vertical components.

The relative positions of currents \( i_1 \) and \( i_2 \) are shown in the diagram below.

Total horizontal component, \( H = 25 \cos 0^\circ - 15 \cos 60^\circ = 17.50 \) \( \text{ (since} \frac{\pi}{3} \text{ rad} = 60^\circ) \)

Total vertical component, \( V = 25 \sin 0^\circ - 15 \sin 60^\circ = -12.99 \)

By Pythagoras, the resultant, \( i_r = \sqrt{17.50^2 + 12.99^2} = 21.79 \)

Phase angle, \( \phi = \tan^{-1} \left( \frac{-12.99}{17.50} \right) = -36.59^\circ \) or -0.639 rad

Hence, by using horizontal and vertical components
\[
i = 25 \sin \omega t - 15 \sin \left( \omega t + \frac{\pi}{3} \right) = 21.79 \sin (\omega t - 0.639)
\]

4. Express \( x = 9 \sin \left( \omega t + \frac{\pi}{3} \right) - 7 \sin \left( \omega t - \frac{3\pi}{8} \right) \) in the form \( A \sin(\omega t \pm \alpha) \) by horizontal and vertical components.

\[
\frac{\pi}{3} \text{ rad} = \frac{\pi}{3} \times \frac{180^\circ}{\pi} = 60^\circ \text{ and } \frac{3\pi}{8} \text{ rad} = \frac{3\pi}{8} \times \frac{180^\circ}{\pi} = 67.5^\circ
\]
The relative positions of currents $x_1$ and $x_2$ are shown in the diagram below.

![Diagram of currents $x_1$ and $x_2$](image)

Total horizontal component, $H = 9 \cos 60^\circ - 7 \cos(-67.5^\circ) = 1.821$

Total vertical component, $V = 9 \sin 60^\circ - 7 \sin(-67.5^\circ) = 14.261$

By Pythagoras, the resultant, $i_r = \sqrt{[1.821]^2 + [14.261]^2} = 14.38$

Phase angle, $\phi = \tan^{-1} \left( \frac{14.261}{1.821} \right) = 82.72^\circ$ or $1.444$ rad

Hence, by using horizontal and vertical components,

$$x = 9 \sin \left( \omega t + \frac{\pi}{3} \right) - 7 \sin \left( \omega t - \frac{3\pi}{8} \right) = 14.38 \sin(\omega t + 1.444)$$

5. The voltage drops across two components when connected in series across an a.c. supply are:

$v_1 = 200 \sin 314.2t$ and $v_2 = 120 \sin \left( 314.2t - \frac{\pi}{5} \right)$ volts, respectively. Determine the

(a) voltage of the supply (given by $v_1 + v_2$) in the form $A \sin(\omega t \pm \alpha)$, and

(b) frequency of the supply.

(a) Total horizontal component, $H = 200 \cos 0^\circ + 120 \cos(-36^\circ) = 297.082$

(since $\frac{\pi}{5} \text{ rad} = \frac{180^\circ}{5} = 36^\circ$)

Total vertical component, $V = 200 \sin 0^\circ + 120 \sin(-36^\circ) = -70.534$

By Pythagoras, the resultant, $i_r = \sqrt{[297.082]^2 + [70.534]^2} = 305.3$ V

Phase angle, $\phi = \tan^{-1} \left( \frac{-70.534}{297.082} \right) = -13.36^\circ$ or $-0.233$ rad

Hence, by using horizontal and vertical components,
\[ v_1 + v_2 = 200 \sin 314.2t + 120 \sin (314.2t - \pi/5) = 305.3 \sin(314.2t - 0.233) \] volts

(b) Angular velocity, \( \omega = 314.2 \text{ rad/s} = 2 \pi f \)

from which, frequency, \( f = \frac{314.2}{2\pi} = 50 \text{ Hz} \)

6. If the supply to a circuit is \( v = 20 \sin 628.3t \) volts and the voltage drop across one of the components is \( v_1 = 15 \sin (628.3t - 0.52) \) volts, calculate the:

(a) voltage drop across the remainder of the circuit, given by \( v - v_1 \), in the form \( A \sin(\omega t \pm \alpha) \)

(b) supply frequency

(c) periodic time of the supply.

(a) \( v - v_1 = 20 \sin 628.3t - 15 \sin (628.3t - 0.52) \)

Total horizontal component, \( H = 20 \cos 0 - 15 \cos(-0.52) = 6.9827 \) (Remember – radians)

Total vertical component, \( V = 20 \sin 0 - 15 \sin(-0.52) = 7.4532 \)

By Pythagoras, the resultant, \( i_R = \sqrt{6.9827^2 + 7.4532^2} = 10.21 \) V

Phase angle, \( \phi = \tan^{-1} \left( \frac{7.4532}{6.9827} \right) = 0.818 \text{ rad} \)

Hence, by using horizontal and vertical components,

\( v - v_1 = 20 \sin 628.3t - 15 \sin (628.3t - 0.52) = 10.21 \sin(628.3t + 0.818) \) volts

(b) Angular velocity, \( \omega = 628.3 \text{ rad/s} = 2 \pi f \)

from which, frequency, \( f = \frac{628.3}{2\pi} = 100 \text{ Hz} \)

(c) Periodic time, \( T = \frac{1}{f} = \frac{1}{100} = 0.01 \text{ s} = 10 \text{ ms} \)

7. The voltages across three components in a series circuit when connected across an a.c. supply are: \( v_1 = 25 \sin \left(300\pi t + \frac{\pi}{6} \right) \) volts, \( v_2 = 40 \sin \left(300\pi t - \frac{\pi}{4} \right) \) volts and
\[ v_3 = 50 \sin \left( 300\pi t + \frac{\pi}{3} \right) \text{ volts.} \]

Calculate the: (a) supply voltage, in sinusoidal form, in the form \( A \sin(\omega t \pm \alpha) \)

(b) frequency of the supply

(c) periodic time

(a) Total horizontal component, \( H = 25 \cos 30^\circ + 40 \cos(-45^\circ) + 50 \cos 60^\circ = 74.935 \)

Total vertical component, \( V = 25 \sin 30^\circ + 40 \sin(-45^\circ) + 50 \sin 60^\circ = 27.517 \)

By Pythagoras, the resultant, \( v_1 + v_2 + v_3 = \sqrt{74.935^2 + 27.517^2} = 79.83 \text{ V} \)

Phase angle, \( \phi = \tan^{-1} \left( \frac{27.517}{74.935} \right) = 20.16^\circ \text{ or } 0.352 \text{ rad} \)

Hence, by using horizontal and vertical components,

supply voltage, \( v_1 + v_2 + v_3 = 79.83 \sin(300\pi t + 0.352) \)

(b) Angular velocity, \( \omega = 300 \pi \text{ rad/s} = 2\pi f \)

from which, frequency, \( f = \frac{300\pi}{2\pi} = 150 \text{ Hz} \)

(c) Periodic time, \( T = \frac{1}{f} = \frac{1}{150} = 0.006667 \text{ s} = 6.667 \text{ ms} \)

8. In an electrical circuit, two components are connected in series. The voltage across the first component is given by \( 80 \sin(\omega t + \pi/3) \) volts, and the voltage across the second component is given by \( 150 \sin(\omega t - \pi/4) \) volts. Determine the total supply voltage to the two components. Give the answer in sinusoidal form.

Total horizontal component, \( H = 80 \cos 60^\circ + 150 \cos(-45^\circ) = 146.066 \)

(since \( \frac{\pi}{3} \text{ rad} = \frac{180^\circ}{3} = 60^\circ \) and \( \frac{\pi}{4} \text{ rad} = \frac{180^\circ}{4} = 45^\circ \))

Total vertical component, \( V = 80 \sin 60^\circ + 150 \sin(-45^\circ) = -36.784 \)

By Pythagoras, the resultant, \( i_R = \sqrt{146.066^2 + 36.784^2} = 150.6 \text{ V} \)
Phase angle, $\phi = \tan^{-1}\left(\frac{-36.784}{146.066}\right) = -14.135^\circ$ or $-0.247$ rad

Hence, by using horizontal and vertical components,

$$80 \sin(\omega t + \pi/3) + 150 \sin(\omega t - \pi/4) = 150.6 \sin(\omega t - 0.247) \text{ volts}$$
EXERCISE 215 Page 584

1. Express \( 8 \sin \omega t + 5 \sin \left( \omega t + \frac{\pi}{4} \right) \) in the form \( A \sin(\omega t \pm \alpha) \) by using complex numbers.

Using complex numbers, \( 8 \sin \omega t + 5 \sin \left( \omega t + \frac{\pi}{4} \right) \equiv 8 \angle 0^\circ + 5 \angle 45^\circ \) in polar form

\[
= (8 + j0) + (3.536 + j3.536) \\
= 11.536 + j3.536 \\
= 12.07 \angle 17.04^\circ = 12.07 \angle 0.297 \text{ rad}
\]

Hence, in sinusoidal form, \( 8 \sin \omega t + 5 \sin \left( \omega t + \frac{\pi}{4} \right) = 12.07 \sin(\omega t + 0.297) \)

2. Express \( 6 \sin \omega t + 9 \sin \left( \omega t - \frac{\pi}{6} \right) \) in the form \( A \sin(\omega t \pm \alpha) \) by using complex numbers.

Using complex numbers, \( 6 \sin \omega t + 9 \sin \left( \omega t - \frac{\pi}{6} \right) \equiv 6 \angle 0^\circ + 9 \angle -30^\circ \) in polar form

\[
(\text{since} \quad \frac{\pi}{6} \text{ rad} = \frac{180^\circ}{6})
\]

\[
= (6 + j0) + (7.794 - j4.500) \\
= 13.794 - j4.500 \\
= 14.51 \angle -18.068^\circ = 14.51 \angle -0.315 \text{ rad}
\]

Hence, in sinusoidal form, \( 6 \sin \omega t + 9 \sin \left( \omega t - \frac{\pi}{6} \right) = 14.51 \sin(\omega t - 0.315) \)

3. Express \( v = 12 \sin \omega t - 5 \sin \left( \omega t - \frac{\pi}{4} \right) \) in the form \( A \sin(\omega t \pm \alpha) \) by using complex numbers.

Using complex numbers, \( 12 \sin \omega t - 5 \sin \left( \omega t + \frac{\pi}{4} \right) \equiv 12 \angle 0^\circ - 5 \angle -45^\circ \) in polar form

\[
= (12 + j0) - (3.536 - j3.536) \\
= 8.464 - j3.536
\]
Hence, in sinusoidal form, \(12 \sin \omega t - 5 \sin \left( \omega t - \frac{\pi}{4} \right) = 9.173 \sin (\omega t - 0.396)\)

4. Express \(x = 10 \sin \left( \omega t + \frac{\pi}{3} \right) - 8 \sin \left( \omega t - \frac{3\pi}{8} \right)\) in the form \(A \sin (\omega t \pm \alpha)\) by using complex numbers.

\[
\frac{\pi}{3} \text{ rad} = \frac{\pi}{3} \times \frac{180^\circ}{\pi} = 60^\circ \quad \text{and} \quad \frac{3\pi}{8} \text{ rad} = \frac{3\pi}{8} \times \frac{180^\circ}{\pi} = 67.5^\circ
\]

Using complex numbers, \(10 \sin \left( \omega t + \frac{\pi}{3} \right) - 8 \sin \left( \omega t - \frac{3\pi}{8} \right) \equiv 10 \angle 60^\circ - 8 \angle -67.5^\circ \) in polar form

\[
= (5 + j8.660) - (3.061 - j7.391)
\]

\[
= 1.939 + j16.051
\]

\[
= 16.168 \angle 83.11^\circ = 16.168 \angle 1.451 \text{ rad}
\]

Hence, in sinusoidal form, \(10 \sin \left( \omega t + \frac{\pi}{3} \right) - 8 \sin \left( \omega t - \frac{3\pi}{8} \right) = 16.168 \sin (\omega t + 1.451)\)

5. The voltage drops across two components when connected in series across an a.c. supply are:

\(v_1 = 240 \sin 314.2t\) and \(v_2 = 150 \sin (314.2t - \pi/5)\) volts, respectively. Determine the

(a) voltage of the supply (given by \(v_1 + v_2\)) in the form \(A \sin (\omega t \pm \alpha)\)

(b) frequency of the supply.

(a) Using complex numbers, \(v_1 + v_2 = 240 \sin 314.2t + 150 \sin (314.2t - \pi/5)\)

\[
\equiv 240 \angle 0^\circ + 150 \angle -36^\circ \quad \text{in polar form (since} \quad \frac{\pi}{5} \text{ rad} = \frac{180^\circ}{5} = 36^\circ)\]

\[
= (240 + j0) + (121.353 - j88.168)
\]

\[
= 361.353 - j88.168
\]

\[
= 371.95 \angle -13.71^\circ = 371.95 \angle -0.239 \text{ rad}
\]

Hence, in sinusoidal form, supply voltage, \(v_1 + v_2 = 371.95 \sin (314.2t - 0.239) \text{ V}\)

(b) Angular velocity, \(\omega = 314.2 \text{ rad/s} = 2\pi f\)
from which, frequency, \( f = \frac{314.2}{2\pi} = 50 \text{ Hz} \)

6. If the supply to a circuit is \( v = 25 \sin 200\pi t \) volts and the voltage drop across one of the components is \( v_1 = 18 \sin(200\pi t - 0.43) \) volts, calculate the:

(a) voltage drop across the remainder of the circuit, given by \( v - v_1 \), in the form \( A \sin(\omega t \pm \alpha) \)

(b) supply frequency

(c) periodic time of the supply.

(a) Using complex numbers, \( v - v_2 = 25 \sin 200\pi t - 18 \sin (200\pi t - 0.43) \)

\[ = 25 \angle 0^\circ - 18 \angle -0.43 \text{ rad in polar form} \]

\[ = (25 + j0) - (16.361 - j7.504) \]

\[ = 8.639 + j7.504 \]

\[ = 11.44 \angle 0.715 \text{ rad} \]

Hence, in sinusoidal form, voltage across remainder of circuit,

\[ v - v_2 = 11.44 \sin(200\pi t + 0.715) \text{ V} \]

(b) Angular velocity, \( \omega = 200\pi \text{ rad/s} = 2\pi f \)

from which, frequency, \( f = \frac{200\pi}{2\pi} = 100 \text{ Hz} \)

(c) Periodic time, \( T = \frac{1}{f} = \frac{1}{100} = 0.010 \text{ s} = 10 \text{ ms} \)

7. The voltages across three components in a series circuit when connected across an a.c. supply are: \( v_1 = 20 \sin \left(300\pi t - \frac{\pi}{6}\right) \) volts, \( v_2 = 30 \sin \left(300\pi t + \frac{\pi}{4}\right) \) volts and \( v_3 = 60 \sin \left(300\pi t - \frac{\pi}{3}\right) \) volts. Calculate the:

(a) supply voltage, in sinusoidal form, in the form \( A \sin(\omega t \pm \alpha) \)

(b) frequency of the supply

(c) periodic time

(d) r.m.s. value of the supply voltage.
(a) Using complex numbers, supply voltage = \( v_1 + v_2 + v_3 \)

\[ \equiv 20\angle -30^\circ + 30\angle 45^\circ + 60\angle -60^\circ \] in polar form

\[ = (17.321 - j10) + (21.213 + j21.213) + (30 - j51.962) \]

\[ = 68.534 - j40.749 \]

\[ = 79.73\angle -30.73^\circ = 79.73\angle -0.536 \text{ rad} \]

Hence, by using complex numbers,

supply voltage, \( v_1 + v_2 + v_3 = 79.73 \sin (300\pi t - 0.536) \)

(b) Angular velocity, \( \omega = 300\pi \text{ rad/s} = 2\pi f \)

from which, frequency, \( f = \frac{300\pi}{2\pi} = 150 \text{ Hz} \)

(c) Periodic time, \( T = \frac{1}{f} = \frac{1}{150} = 0.006667 \text{ s} = 6.667 \text{ ms} \)

(d) R.m.s. value of the supply voltage = \( 0.707 \times 79.73 = 56.37 \text{ V} \)

8. Measurements made at a substation at peak demand of the current in the red, yellow and blue phases of a transmission system are: \( I_{\text{red}} = 1248\angle -15^\circ \text{ A}, \) \( I_{\text{yellow}} = 1120\angle -135^\circ \text{ A} \) and \( I_{\text{blue}} = 1310\angle 95^\circ \text{ A}. \) Determine the current in the neutral cable if the sum of the currents flows through it.

Current in neutral cable = \( I_{\text{red}} + I_{\text{yellow}} + I_{\text{blue}} \)

\[ = 1248\angle -15^\circ + 1120\angle -135^\circ + 1310\angle 95^\circ \]

\[ = (1205.475 - j323.006) + (-791.960 - j791.960) + (-114.174 + j1305.015) \]

\[ = 299.341 + j190.049 \]

\[ = 354.6\angle 32.41^\circ \]

Hence, by using complex numbers,

current in neutral cable = \( 354.6\angle 32.41^\circ \text{ A} \)