CHAPTER 32 REDUCTION OF NON-LINEAR LAWS TO LINEAR FORM

EXERCISE 137 Page 348

1. For the law \( y = d + cx^2 \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.

\( y = d + cx^2 \), i.e. \( y = cx^2 + d \) which compares with \( y = mx + c \), showing that:

(a) \( y \) should be plotted on the vertical axis
(b) \( x^2 \) should be plotted on the horizontal axis
(c) the gradient is \( c \)
(d) the vertical axis intercept is \( d \)

2. For the law \( y - a = b \sqrt{x} \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.

\( y - a = b \sqrt{x} \), i.e. \( y = b \sqrt{x} + a \) which compares with \( y = mx + c \), showing that:

(a) \( y \) should be plotted on the vertical axis
(b) \( \sqrt{x} \) should be plotted on the horizontal axis
(c) the gradient is \( b \)
(d) the vertical axis intercept is \( a \)

3. For the law \( y - e = \frac{f}{x} \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.
\[ y - e = \frac{f}{x} , \text{ i.e. } y = f \left( \frac{1}{x} \right) + e \] which compares with \( y = mx + c \), showing that:

(a) \( y \) should be plotted on the vertical axis

(b) \( \frac{1}{x} \) should be plotted on the horizontal axis

(c) the gradient is \( f \)

(d) the vertical axis intercept is \( e \)

4. For the law \( y - cx = bx^2 \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.

\[ y - cx = bx^2 , \text{ i.e. } y = bx^2 + cx , \text{ i.e. } \frac{y}{x} = bx + c \] which compares with \( y = mx + c \), showing that:

(a) \( \frac{y}{x} \) should be plotted on the vertical axis

(b) \( x \) should be plotted on the horizontal axis

(c) the gradient is \( b \)

(d) the vertical axis intercept is \( c \)

5. For the law \( y = \frac{a}{x} + bx \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.

\[ y = \frac{a}{x} + bx , \text{ i.e. } \frac{y}{x} = a \left( \frac{1}{x^2} \right) + b \] which compares with \( y = mx + c \), showing that:

(a) \( \frac{y}{x} \) should be plotted on the vertical axis

(b) \( \frac{1}{x^2} \) should be plotted on the horizontal axis

(c) the gradient is \( a \)
(d) the vertical axis intercept is $b$

6. In an experiment the resistance of wire is measured for wires of different diameters with the following results.

<table>
<thead>
<tr>
<th>$R$ ohms</th>
<th>1.64</th>
<th>1.14</th>
<th>0.89</th>
<th>0.76</th>
<th>0.63</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ mm</td>
<td>1.10</td>
<td>1.42</td>
<td>1.75</td>
<td>2.04</td>
<td>2.56</td>
</tr>
</tbody>
</table>

It is thought that $R$ is related to $d$ by the law $R = \frac{a}{d^2} + b$, where $a$ and $b$ are constants. Verify this and find the approximate values for $a$ and $b$. Determine the cross-sectional area needed for a resistance reading of 0.50 ohms.

Comparing $R = \frac{a}{d^2} + b$ with $y = mx + c$ shows that $y$ is to be plotted vertically against $\frac{1}{d^2}$ horizontally.

A table of values is drawn up as shown below.

<table>
<thead>
<tr>
<th>$R$ ohms</th>
<th>1.64</th>
<th>1.14</th>
<th>0.89</th>
<th>0.76</th>
<th>0.63</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{d^2}$</td>
<td>0.83</td>
<td>0.50</td>
<td>0.33</td>
<td>0.24</td>
<td>0.15</td>
</tr>
</tbody>
</table>

A graph of $R$ against $\frac{1}{d^2}$ is shown below.
A law of the form $R = \frac{a}{d^2} + b$ is verified since a straight line graph results

Gradient of straight line, $a = \frac{AB}{BC} = \frac{1.9 - 0.4}{1.0 - 0} = \frac{1.5}{1.0} = 1.5$

Vertical axis intercept, $b = 0.4$

Hence, the law of the graph is: $R = \frac{1.5}{d^2} + 0.4$

When $R = 0.50$ ohms, $0.50 = \frac{1.5}{d^2} + 0.4$

from which, $0.50 - 0.4 = \frac{1.5}{d^2}$ and $d^2 = \frac{1.5}{0.10} = 15$ i.e. $d = \sqrt{15}$

Cross-sectional area $= \pi r^2 = \pi \frac{d^2}{4} = \pi \left(\frac{\sqrt{15}}{4}\right)^2 = \pi \left(\frac{15}{4}\right) = 11.78$ mm$^2$

7. Corresponding experimental values of two quantities $x$ and $y$ are given below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.5</th>
<th>3.0</th>
<th>4.5</th>
<th>6.0</th>
<th>7.5</th>
<th>9.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>11.5</td>
<td>25.0</td>
<td>47.5</td>
<td>79.0</td>
<td>119.5</td>
<td>169.0</td>
</tr>
</tbody>
</table>

By plotting a suitable graph verify that $y$ and $x$ are connected by a law of the form $y = kx^2 + c$,

where $k$ and $c$ are constants. Determine the law of the graph and hence find the value of $x$ when $y$ is 60.0

Comparing $y = kx^2 + c$ with $y = mx + c$

shows that $y$ is to be plotted vertically against $x^2$ horizontally

A table of values is drawn up as shown below

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.5</th>
<th>3.0</th>
<th>4.5</th>
<th>6.0</th>
<th>7.5</th>
<th>9.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>2.25</td>
<td>9.0</td>
<td>20.25</td>
<td>36.0</td>
<td>56.25</td>
<td>81.0</td>
</tr>
<tr>
<td>$y$</td>
<td>11.5</td>
<td>25.0</td>
<td>47.5</td>
<td>79.0</td>
<td>119.5</td>
<td>169.0</td>
</tr>
</tbody>
</table>

A graph of $y$ against $x^2$ is shown below

A law of the form $y = kx^2 + c$ is verified since a straight line graph results
Gradient of straight line, \( k = \frac{AB}{BC} = \frac{146 - 26}{70 - 10} = \frac{120}{60} = 2 \)

Vertical axis intercept, \( c = 7 \)

Hence, the law of the graph is: \( y = 2x^2 + 7 \)

When \( y = 60.0, \) \( 60.0 = 2x^2 + 7 \) i.e. \( 60.0 - 7 = 2x^2 \) and \( x = \sqrt{\frac{53.0}{2}} = 5.15 \)

8. Experimental results of the safe load \( L \) kN, applied to girders of varying spans, \( d \) m, are shown below.

| Span, \( d \) m | 2.0 | 2.8 | 3.6 | 4.2 | 4.8 |
| Load, \( L \) kN | 475 | 339 | 264 | 226 | 198 |

It is believed that the relationship between load and span is \( L = \frac{c}{d} \), where \( c \) is a constant.

Determine (a) the value of constant \( c \) and (b) the safe load for a span of 3.0 m

Comparing \( L = \frac{c}{d} \) with \( y = mx + c \)

shows that \( L \) is to be plotted vertically against \( \frac{1}{d} \) horizontally.

A table of values is drawn up as shown below

<table>
<thead>
<tr>
<th>Load, ( L ) kN</th>
<th>475</th>
<th>339</th>
<th>264</th>
<th>226</th>
<th>198</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{d} )</td>
<td>0.50</td>
<td>0.36</td>
<td>0.28</td>
<td>0.24</td>
<td>0.21</td>
</tr>
</tbody>
</table>
A graph of $L$ against $\frac{1}{d}$ is shown below

(a) A law of the form $L = \frac{c}{d}$ is verified since a straight line graph results

Gradient of straight line, $c = \frac{AB}{BC} = \frac{475 - 0}{0.5 - 0} = \frac{475}{0.5} = 950$

Hence, the law of the graph is: $L = \frac{950}{d}$

(b) When span, $d = 3.0$ m, the safe load, $L = \frac{950}{3.0} = 317$ kN

9. The following results give corresponding values of two quantities $x$ and $y$ which are believed to be related by a law of the form $y = ax^2 + bx$ where $a$ and $b$ are constants.

<table>
<thead>
<tr>
<th>$x$</th>
<th>3.4</th>
<th>5.2</th>
<th>6.5</th>
<th>7.3</th>
<th>9.1</th>
<th>12.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>33.86</td>
<td>55.54</td>
<td>72.80</td>
<td>84.10</td>
<td>111.4</td>
<td>168.1</td>
</tr>
</tbody>
</table>

Verify the law and determine approximate values of $a$ and $b$. Hence determine (i) the value of $y$ when $x$ is 8.0 and (ii) the value of $x$ when $y$ is 146.5

Comparing $y = ax^2 + bx$, i.e. $\frac{y}{x} = ax + b$ with $y = mx + c$ where $a$ is the gradient and $b$ the
vertical axis intercept shows that \( \frac{y}{x} \) is to be plotted vertically against \( x \) horizontally

A table of values is drawn up as shown below

\[
\begin{array}{cccccc}
 y & 33.86 & 55.54 & 72.80 & 84.10 & 111.4 & 168.1 \\
 x & 3.4 & 5.2 & 6.5 & 7.3 & 9.1 & 12.4 \\
 \frac{y}{x} & 9.96 & 10.68 & 11.20 & 11.52 & 12.24 & 13.56 \\
\end{array}
\]

A graph of \( \frac{y}{x} \) against \( x \) is shown below

Gradient of straight line, \( a = \frac{AB}{BC} = \frac{13.4 - 9.4}{12 - 2} = \frac{4}{10} = 0.4 \), hence, \( a = 0.4 \)

Vertical axis intercept, \( b = 8.6 \)

(i) \( \frac{y}{x} = ax + b \) hence when \( x = 8.0 \), then \( \frac{y}{8.0} = 0.4(8.0) + 8.6 = 11.8 \)
Thus, \( y = 8.0(11.8) = 94.4 \)

(ii) \( \frac{y}{x} = ax + b \) hence when \( y = 146.5 \), \( \frac{146.5}{x} = 0.4x + 8.6 \)

Thus, \( 146.5 = 0.4x^2 + 8.6x \)

and \( 0.4x^2 + 8.6x - 146.5 = 0 \)

from which, \( x = \frac{-8.6 \pm \sqrt{[8.6^2 - 4(0.4)(-146.5)]}}{2(0.4)} = \frac{-8.6 \pm \sqrt{308.36}}{0.8} \)

i.e. \( x = 11.2 \) (or \(-32.7\), which is neglected)
1. For the law \( y = ba^x \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.

\[
y = ba^x \quad \text{and taking logarithms of both sides gives}
\]
\[
\log y = \log (ba^x) = \log b + \log a^x
\]
i.e. \( \log y = x \log a + \log b \) which compares with \( y = mx + c \), showing that:

(a) \( \log y \) should be plotted on the vertical axis

(b) \( x \) should be plotted on the horizontal axis

(c) the gradient is \( \log a \)

(d) the vertical axis intercept is \( \log b \)

2. For the law \( y = kx^L \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.

\[
y = kx^L \quad \text{and taking logarithms of both sides gives:}
\]
\[
\log y = \log (kx^L) = \log k + \log x^L
\]
i.e. \( \log y = L \log x + \log k \) which compares with \( y = mx + c \), showing that:

(a) \( \log y \) should be plotted on the vertical axis

(b) \( \log x \) should be plotted on the horizontal axis

(c) the gradient is \( L \)

(d) the vertical axis intercept is \( \log k \)

3. For the law \( \frac{y}{m} = e^{ix} \) to be verified it is necessary to plot a graph of the variables in a modified form. State (a) what should be plotted on the vertical axis, (b) what should be plotted on the horizontal axis, (c) the gradient and (d) the vertical axis intercept.
\[ \frac{y}{m} = e^{mx}, \text{ i.e. } y = m e^{mx} \] and taking logarithms of both sides give:

\[ \ln y = \ln (me^{mx}) = \ln m + \ln e^{mx} \]

i.e. \( \ln y = n \ln x + \ln m \) which compares with \( y = mx + c \), showing that:

(a) \( \ln y \) should be plotted on the vertical axis
(b) \( x \) should be plotted on the horizontal axis
(c) the gradient is \( n \)
(d) the vertical axis intercept is \( \ln m \)

4. The luminosity \( I \) of a lamp varies with the applied voltage \( V \) and the relationship between \( I \) and \( V \) is thought to be \( I = kV^n \). Experimental results obtained are:

| \( I \) candelas | 1.92 | 4.32 | 9.72 | 15.87 | 23.52 | 30.72 |
| \( V \) volts   | 40   | 60   | 90   | 115   | 140   | 160   |

Verify that the law is true and determine the law of the graph. Determine also the luminosity when 75 V is applied across the lamp.

\[ I = kV^n \] and taking logarithms of both sides gives:

\[ \lg I = \lg (kV^n) = \lg k + \lg V^n \]

i.e. \( \lg I = n \lg V + \lg k \) which compares with \( y = mx + c \)

Drawing up a table of values gives:

| \( \lg I \) | 0.28 | 0.64 | 0.99 | 1.20 | 1.37 | 1.49 |
| \( \lg V \) | 1.60 | 1.78 | 1.95 | 2.06 | 2.15 | 2.20 |

A graph of \( \lg I \) against \( \lg V \) is shown below.

Gradient of straight line \( \frac{AB}{BC} = \frac{1.49 - 0.28}{2.20 - 1.60} = \frac{1.21}{0.6} = 2 \), hence, \( n = 2 \)

Taking point A on the graph, \( 1.49 = n(2.20) + \lg k \)

i.e. \( 1.49 = 2(2.20) + \lg k \)
from which, \( \lg k = 1.49 - 4.40 = -2.91 \)
and \( k = 10^{-2.91} = 0.0012 \)

Hence, \( I = kV^n \) i.e. \( I = 0.0012V^2 \)

When \( V = 75 \text{ V} \), \( I = 0.0012(75)^2 = 6.75 \text{ candelas} \)

5. The head of pressure \( h \) and the flow velocity \( v \) are measured and are believed to be connected by the law \( v = ah^b \), where \( a \) and \( b \) are constants.

The results are as shown below.

<table>
<thead>
<tr>
<th>( h )</th>
<th>10.6</th>
<th>13.4</th>
<th>17.2</th>
<th>24.6</th>
<th>29.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>9.77</td>
<td>11.0</td>
<td>12.44</td>
<td>14.88</td>
<td>16.24</td>
</tr>
</tbody>
</table>

Verify that the law is true and determine values of \( a \) and \( b \).

\( v = ah^b \) and taking logarithms of both sides gives:

\[ \lg v = \lg (ah^b) = \lg a + \lg h^b \]
i.e. \( \lg v = b \lg h + \lg a \) which compares with \( y = mx + c \)

Drawing up a table of values gives:

\[
\begin{array}{cccccc}
\lg v & 0.99 & 1.04 & 1.09 & 1.17 & 1.21 \\
\lg h & 1.03 & 1.13 & 1.24 & 1.39 & 1.47 \\
\end{array}
\]

A graph of \( \lg v \) against \( \lg h \) is shown below.

Since a straight line results the law is verified

Gradient of straight line, \( b = \frac{AB}{BC} = \frac{1.48 - 0.48}{2.0 - 0} = \frac{1.0}{2.0} = 0.5 \)

Vertical axis intercept, \( \lg a = 0.48 \) from which, \( a = 10^{0.48} = 3.0 \)

6. Experimental values of \( x \) and \( y \) are measured as follows.

\[
\begin{array}{cccccc}
x & 0.4 & 0.9 & 1.2 & 2.3 & 3.8 \\
y & 8.35 & 13.47 & 17.94 & 51.32 & 215.20 \\
\end{array}
\]

The law relating \( x \) and \( y \) is believed to be of the form \( y = ab^x \), where \( a \) and \( b \) are constants.

Determine the approximate values of \( a \) and \( b \). Hence find the value of \( y \) when \( x = 2.0 \) and the value of \( x \) when \( y = 100 \)

\[
y = ab^x \text{ and taking logarithms of both sides gives:} \\
\lg y = \lg(ab^x) = \lg a + \lg b^x 
\]
i.e. \( \lg y = x \lg b + \lg a \) which compares with \( y = mx + c \)

Drawing up a table of values gives:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.4</th>
<th>0.9</th>
<th>1.2</th>
<th>2.3</th>
<th>3.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8.35</td>
<td>13.47</td>
<td>17.94</td>
<td>51.32</td>
<td>215.20</td>
</tr>
<tr>
<td>( \lg y )</td>
<td>0.92</td>
<td>1.13</td>
<td>1.25</td>
<td>1.71</td>
<td>2.33</td>
</tr>
</tbody>
</table>

A graph of \( \lg y \) against \( x \) is shown below.

Gradient of straight line \( \frac{AB}{BC} = \frac{2.33-0.92}{3.8-0.4} = \frac{1.41}{3.4} = 0.4147 = \lg b \), hence, \( b = 10^{0.4147} = 2.6 \)

Vertical axis intercept, \( \lg a = 0.75 \), hence, \( a = 10^{0.75} = 5.6 \)

Hence, \( y = ab^x \) i.e. \( y = 5.6(2.6)^x \)

When \( x = 2.0 \), \( y = 5.6(2.6)^2 = 37.86 \)

When \( y = 100 \), \( 100 = 5.6(2.6)^x \) i.e. \( \frac{100}{5.6} = 2.6^x \)

i.e. \( \lg \left( \frac{100}{5.6} \right) = \lg 2.6^x = x \lg 2.6 \) and \( x = \frac{\lg \left( \frac{100}{5.6} \right)}{\lg 2.6} = 3.0 \)
7. The activity of a mixture of radioactive isotope is believed to vary according to the law $R = R_0 t^{-c}$, where $R_0$ and $c$ are constants.

Experimental results are shown below.

<table>
<thead>
<tr>
<th>$R$</th>
<th>9.72</th>
<th>2.65</th>
<th>1.15</th>
<th>0.47</th>
<th>0.32</th>
<th>0.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>2</td>
<td>5</td>
<td>9</td>
<td>17</td>
<td>22</td>
<td>28</td>
</tr>
</tbody>
</table>

Verify that the law is true and determine approximate values of $R_0$ and $c$.

$R = R_0 t^{-c}$ and taking logarithms of both sides gives:

$$\lg R = \lg \left( R_0 t^{-c} \right) = \lg R_0 + \lg t^{-c}$$

i.e. $\lg R = -c \lg t + \lg R_0$ which compares with $y = mx + c$

Drawing up a table of values gives:

| $\lg R$ | 0.99 | 0.42 | 0.06 | –0.33 | –0.50 | –0.64 |
| $\lg t$ | 0.30 | 0.70 | 0.95 | 1.23  | 1.34  | 1.45  |

A graph of $\lg R$ against $\lg t$ is shown below.

The law $R = R_0 t^{-c}$ is true since the graph is a straight line.

Vertical axis intercept = $\ln R_0 = 1.4$ hence, $R_0 = 10^{1.4} = 25.1$

Gradient of straight line = \frac{AB}{BC} = \frac{0.99 - 0.30}{0.30 - 1.45} = \frac{1.63}{-1.15} = -1.42 = -c$ i.e. $c = 1.42$
8. Determine the law of the form \( y = ae^{kx} \) which relates the following values.

| \( x \) | \(-4.0\) | 5.3 | 9.8 | 17.4 | 32.0 | 40.0 |
| \( y \) | 0.0306 | 0.285 | 0.841 | 5.21 | 173.2 | 1181 |

Taking logarithms to base \( e \) of both sides of \( y = ae^{kx} \) gives:

\[
\ln y = \ln (ae^{kx}) = \ln a + \ln e^{kx}
\]

i.e. \( \ln y = \ln a + kx \) i.e. \( \ln y = kx + \ln a \)

Hence, if \( \ln y \) is plotted vertically against \( x \) horizontally, a straight-line graph should result with gradient \( k \) and vertical axis intercept \( \ln a \)

A table of values is produced as shown below:

| \( x \) | \(-4.0\) | 5.3 | 9.8 | 17.4 | 32.0 | 40.0 |
| \( \ln y \) | \(-3.49\) | \(-1.26\) | \(-0.17\) | 1.65 | 5.15 | 7.07 |

A graph of \( \ln y \) against \( x \) is shown below.

Gradient, \( k = \frac{AB}{BC} = \frac{7.07 - (-2.50)}{40 - 0} = \frac{9.57}{40} = 0.24 \)

Vertical intercept, \( \ln a = -2.5 \) from which, \( a = e^{-2.5} = 0.08 \)

Hence, the law of the graph is: \( y = ae^{kx} \) i.e. \( y = 0.08e^{0.24x} \)
9. The tension $T$ in a belt passing round a pulley wheel and in contact with the pulley over an angle of $\theta$ radians is given by $T = T_0 e^{\mu \theta}$, where $T_0$ and $\mu$ are constants. Experimental results obtained are:

\[
\begin{array}{cccccc}
T \text{ newtons} & 47.9 & 52.8 & 60.3 & 70.1 & 80.9 \\
\theta \text{ radians} & 1.12 & 1.48 & 1.97 & 2.53 & 3.06 \\
\end{array}
\]

Determine approximate values of $T_0$ and $\mu$. Hence find the tension when $\theta$ is 2.25 radians and the value of $\theta$ when the tension is 50.0 newtons.

Taking logarithms to base $e$ of both sides of $T = T_0 e^{\mu \theta}$ gives:

\[
\ln T = \ln (T_0 e^{\mu \theta}) = \ln T_0 + \ln e^{\mu \theta}
\]

i.e. \( \ln T = \ln T_0 + \mu \theta \) i.e. \( \ln T = \mu \theta + \ln T_0 \)

Hence, if \( \ln T \) is plotted vertically against $\theta$ horizontally, a straight-line graph should result with gradient $\mu$ and vertical axis intercept $\ln T_0$.

A table of values is produced as shown below

\[
\begin{array}{cccccc}
\theta & 1.12 & 1.48 & 1.97 & 2.53 & 3.06 \\
\ln T & 3.87 & 3.97 & 4.10 & 4.25 & 4.39 \\
\end{array}
\]

A graph of $\ln T$ against $\theta$ is shown below
Gradient, \( \mu = \frac{AB}{BC} = \frac{4.39 - 3.87}{3.06 - 1.12} = \frac{0.52}{1.94} = 0.27 \)

Taking point A, (4.39, 3.06), \( \ln T = \mu \theta + \ln T_0 \)
i.e. \( 4.39 = (0.27)(3.06) + \ln T_0 \)
from which, \( \ln T_0 = 4.39 - (0.27)(3.06) = 3.5638 \)
and \( T_0 = e^{3.5638} = 35.3 \text{ N} \)
When \( \theta = 2.25 \), \( T = T_0 e^{\mu \theta} = 35.3 e^{(0.27)(2.25)} = 64.8 \text{ N} \)
When \( T = 50.0 \text{ N} \), \( 50.0 = 35.3 e^{0.27 \theta} \)
i.e. \( \frac{50.0}{36.4} = e^{0.27 \theta} \) and \( \ln \left( \frac{50.0}{35.3} \right) = \ln \left( e^{0.27 \theta} \right) = 0.27 \theta \)
and \( \theta = \frac{1}{0.27} \ln \left( \frac{50.0}{35.3} \right) = 1.29 \text{ radians} \)