CHAPTER 19 NUMBER SEQUENCES

EXERCISE 77 Page 167

1. Determine the next two terms in the series: 5, 9, 13, 17, ...

It is noticed that the sequence 5, 9, 13, 17, ... progressively increases by 4, thus the next two terms will be 21 and 25

2. Determine the next two terms in the series: 3, 6, 12, 24, ...

It is noticed that the sequence 3, 6, 12, 24, ... progressively doubles, thus the next two terms will be 48 and 96

3. Determine the next two terms in the series: 112, 56, 28, ...

It is noticed that the sequence 112, 56, 28, ... progressively halves, thus the next two terms will be 14 and 7

4. Determine the next two terms in the series: 12, 7, 2, ...

It is noticed that the sequence 12, 7, 2, ... progressively decreases by 5, thus the next two terms will be –3 and –8

5. Determine the next two terms in the series: 2, 5, 10, 17, 26, 37, ...

It is noticed that the sequence 2, 5, 10, 17, 26, 37, ... progressively increases by 3, then 5, then 7, then 9, and so on. Thus the next two terms will be 37 + 13 = 50 and 50 + 15 = 65

6. Determine the next two terms in the series: 1, 0.1, 0.01, ...

It is noticed that the sequence 1, 0.1, 0.01, ... progressively decreases to one tenth of its previous value, thus the next two terms will be 0.001 and 0.0001
7. Determine the next two terms in the series: 4, 9, 19, 34, …

It is noticed that the sequence 4, 9, 19, 34, … progressively increases by 5, then 10, then 15, and so on. Thus the next two terms will be $34 + 20 = 54$ and $54 + 25 = 79$.
EXERCISE 78 Page 168

1. The \( n \)th term of a sequence is given by \( 2n - 1 \). Write down the first four terms.

When \( n = 1 \), \( 2n - 1 = 1 \)  
When \( n = 2 \), \( 2n - 1 = 3 \)
When \( n = 3 \), \( 2n - 1 = 5 \)  
When \( n = 4 \), \( 2n - 1 = 7 \)

Hence, the first four terms are 1, 3, 5, 7, ...

2. The \( n \)th term of a sequence is given by \( 3n + 4 \). Write down the first five terms.

When \( n = 1 \), \( 3n + 4 = 7 \)  
When \( n = 2 \), \( 3n + 4 = 10 \)
When \( n = 3 \), \( 3n + 4 = 13 \)  
When \( n = 4 \), \( 3n + 4 = 16 \)
When \( n = 5 \), \( 3n + 4 = 19 \)

Hence, the first five terms are 7, 10, 13, 16, 19

3. Write down the first four terms of the sequence given by \( 5n + 1 \)

The first four terms of the series \( 5n + 1 \) will be:

\[ 5(1) + 1, \quad 5(2) + 1, \quad 5(3) + 1 \quad \text{and} \quad 5(4) + 1 \]

i.e. 6, 11, 16 and 21

4. Find the \( n \)th term in the series: 5, 10, 15, 20, …

We notice that the gap between each of the given four terms is 5, hence the law relating the numbers is: ‘\( 5n + \) something’

The second term, \( 10 = 5n + \) something,

so when \( n = 2 \), then \( 10 = 10 + \) something,

so the ‘something’ must be 0

Thus, the \( n \)th term of the series 5, 10, 15, 20, … is: \( 5n \)

5. Find the \( n \)th term in the series: 4, 10, 16, 22, …
We notice that the gap between each of the given four terms is 6, hence the law relating the numbers is: ‘$6n + \text{something}$’

The second term, $\quad 10 = 6n + \text{something}$,

so when $n = 2$, then $\quad 10 = 12 + \text{something}$,

so the ‘something’ must be $-2$ (from simple equations)

Thus, the $n$th term of the series $4, 10, 16, 22, \ldots$ is: $6n - 2$

6. Find the $n$th term in the series: $3, 5, 7, 9, \ldots$

We notice that the gap between each of the given four terms is 2, hence the law relating the numbers is: ‘$2n + \text{something}$’

The second term, $\quad 5 = 2n + \text{something}$,

so when $n = 2$, then $\quad 5 = 4 + \text{something}$,

so the ‘something’ must be 1 (from simple equations)

Thus, the $n$th term of the series $3, 5, 7, 9, \ldots$ is: $2n + 1$

7. Find the $n$th term in the series: $2, 6, 10, 14, \ldots$

We notice that the gap between each of the given four terms is 4, hence the law relating the numbers is: ‘$4n + \text{something}$’

The second term, $\quad 6 = 4n + \text{something}$,

so when $n = 2$, then $\quad 6 = 8 + \text{something}$,

so the ‘something’ must be $-2$ (from simple equations)

Thus, the $n$th term of the series $2, 6, 10, 14, \ldots$ is: $4n - 2$

8. Find the $n$th term in the series: $9, 12, 15, 18, \ldots$

We notice that the gap between each of the given four terms is 3, hence the law relating the numbers is: ‘$3n + \text{something}$’
The second term, $12 = 3n + \text{something}$,
so when $n = 2$, then $12 = 6 + \text{something}$,
so the ‘something’ must be $6$ (from simple equations)
Thus, the $n$th term of the series $9, 12, 15, 18, \ldots$ is: $3n + 6$

9. Write down the next two terms of the series: $1, 8, 27, 64, 125, \ldots$

This is a special series and does not follow the pattern of the previous examples. Each of the terms in the given series are cubic numbers,
i.e. $1, 8, 27, 64, 125, \ldots \equiv 1^3, 2^3, 3^3, 4^3, 5^3, \ldots$
Hence the $n$th term is: $n^3$
Thus, the 6th term is: $6^3 = 216$ and the 7th term is: $7^3 = 343$
1. Find the 11th term of the series 8, 14, 20, 26, ...

The 11th term of the series 8, 14, 20, 26, … is given by:

\[ a + (n - 1)d \]  \quad \text{where} \quad a = 8, \quad n = 11 \quad \text{and} \quad d = 6

Hence, the 11th term is: \( 8 + (11 - 1)(6) = 8 + 60 = 68 \)

2. Find the 17th term of the series 11, 10.7, 10.4, 10.1, ...

The 17th term of the series 11, 10.7, 10.4, 10.1, … is given by:

\[ a + (n - 1)d \]  \quad \text{where} \quad a = 11, \quad n = 17 \quad \text{and} \quad d = -0.3

Hence, the 17th term is: \( 11 + (17 - 1)(-0.3) = 11 - 4.8 = 6.2 \)

3. The seventh term of a series is 29 and the eleventh term is 54. Determine the sixteenth term.

The \( n \)th term of an arithmetic progression is: \( a + (n - 1)d \)

The 7th term is: \[ a + 6d = 29 \]  \quad (1)

The 11th term is: \[ a + 10d = 54 \]  \quad (2)

(2) – (1) gives: \[ 4d = 25 \]  \quad \text{from which,} \quad d = \frac{25}{4}

Substituting in (1) gives: \[ a + 6\left(\frac{25}{4}\right) = 29 \]

i.e. \[ a + 37.5 = 29 \]  \quad \text{from which,} \quad a = 29 - 37.5 = -8.5

Hence, the 16th term is: \[ -8.5 + (16 - 1)\left(\frac{25}{4}\right) = -8.5 + 93.75 = 85.25 \]

4. Find the 15th term of an arithmetic progression of which the first term is 2.5 and the tenth term is 16

The \( n \)th term of an arithmetic progression is: \( a + (n - 1)d \) and if \( a = 2.5 \) and the 10th term is 16, then:

\[ 2.5 + 9d = 16 \]
from which, \[ 9d = 16 - 2.5 = 13.5 \]
and \[ d = 13.5/9 = 1.5 \]
Hence, the 15th term is: \[ a + (n - 1)d = 2.5 + (15 - 1)(1.5) \]
\[ = 2.5 + (14)(1.5) = 2.5 + 21 = 23.5 \]

5. Determine the number of the term which is 29 in the series 7, 9.2, 11.4, 13.6, ...

\[ 29 = 7 + (n - 1)(2.2) \]
from which, \[ 29 - 7 = 2.2(n - 1) \]
i.e. \[ \frac{22}{2.2} = n - 1 \]
i.e. \[ 10 = n - 1 \]
and \[ n = 11 \]
i.e. 29 is the 11th term of the series

6. Find the sum of the first 11 terms of the series 4, 7, 10, 13, ...

In the series 4, 7, 10, 13, ... \( a = 4 \) and \( d = 3 \)
Sum of series, \[ S_n = \frac{n}{2} [2a + (n - 1)d] \]
When \( n = 11 \), \[ S_{11} = \frac{11}{2} [2(4) + (11 - 1)(3)] = 5.5[8 + 30] = 209 \]

7. Determine the sum of the series 6.5, 8.0, 9.5, 11.0, …, 32

In the series: 6.5, 8.0, 9.5, 11.0, …, 32, \( a = 6.5 \) and \( d = 1.5 \)
The \( n \)th term is 32, hence, \( a + (n - 1)d = 32 \)
i.e. \( 6.5 + (n - 1)(1.5) = 32 \)
from which, \[ 32 - 6.5 = (n - 1)(1.5) \]
and \[ \frac{25.5}{1.5} = n - 1 \]
i.e. \( 17 = n - 1 \)
and \( n = 18 \)
Sum of series, \[ S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{18}{2} [2(6.5) + (18 - 1)(1.5)] = 9[13 + 25.5] = 346.5 \]
EXERCISE 80 Page 170

1. The sum of 15 terms of an arithmetic progression is 202.5 and the common difference is 2. Find the first term of the series.

\[ n = 15, \ d = 2 \ \text{and} \ S_{15} = 202.5. \]  Since the sum of \( n \) terms of an AP is given by:

\[ S_n = \frac{n}{2} [2a + (n-1)d] \]

Then

\[ 202.5 = \frac{15}{2} [2a + (15-1) \times 2] = \frac{15}{2} [2a + 28] \]

Hence,

\[ \frac{202.5 \times 2}{15} = 2a + 28 \]

i.e.

\[ 27 = 2a + 28 \]

Thus,

\[ 2a = 27 - 28 = -1 \]

from which,

\[ a = \frac{-1}{2} = -0.5 \]

i.e. the first term, \( a = -0.5 \)

2. Three numbers are in arithmetic progression. Their sum is 9 and their product is 20.25. Determine the three numbers.

Let the three numbers be \((a - d), \ a \ \text{and} \ (a + d)\)

Thus,

\[ (a - d) + a + (a + d) = 9 \]

i.e.

\[ 3a = 9 \ \text{and} \ a = 3 \]

Also,

\[ a(a - d)(a + d) = 20.25 \]

Since, \( a = 3 \), then \( 3(9 - d^2) = 20.25 \)

i.e.

\[ 9 - d^2 = \frac{20.25}{3} = 6.75 \]

and \( 9 - 6.75 = d^2 \) from which, \( d^2 = 2.25 \) and \( d = \sqrt{2.25} = 1.5 \)

Hence, the three numbers are: \((a - d) = 3 - 1.5 = 1.5, \ a = 3 \ \text{and} \ (a + d) = 3 + 1.5 = 4.5\)
3. Find the sum of all the numbers between 5 and 250 which are exactly divisible by 4

The series 8, 12, 16, 20, ..., 248 is an AP whose first term \(a = 8\) and common difference \(d = 4\)

The last term is: \(a + (n - 1)d = 248\)

i.e. \(8 + (n - 1)4 = 248\)

Hence, \((n - 1) = \frac{248 - 8}{4} = \frac{240}{4} = 60\) from which, \(n = 61\)

The sum of all 61 terms is given by:

\[
S_{61} = \frac{n}{2}[2a + (n - 1)d] = \frac{61}{2}[2(8) + (61 - 1)4] = \frac{61}{2}[16 + 240] = 30.5(256) = 7808
\]

4. Find the number of terms of the series 5, 8, 11, ... of which the sum is 1025

Sum of \(n\) terms is given by: \(S_n = \frac{n}{2}[2a + (n - 1)d]\)

i.e. \(1025 = \frac{n}{2}[2(5) + (n - 1)(3)]\)

i.e. \(2 \times 1025 = n[10 + 3(n - 1)]\)

Hence, \(2050 = n[10 + 3n - 3] = n[7 + 3n] = 7n + 3n^2\)

i.e. \(3n^2 + 7n - 2050 = 0\)

This is a quadratic equation, hence \(n = \frac{-7 \pm \sqrt{7^2 - 4(3)(-2050)}}{2(3)} = \frac{-7 \pm \sqrt{24649}}{6} = \frac{-7 \pm 157}{6}\)

i.e. number of terms, \(n = 25\) (the negative answer having no meaning)

5. Insert four terms between 5 and 22.5 to form an arithmetic progression.

The \(n\)th term of an arithmetic progression is: \(a + (n - 1)d\) and if \(a = 5\) and the 6th term is 22.5,

then: \(5 + (6 - 1)d = 22.5\)

i.e. \(5d = 22.5 - 5 = 17.5\)

from which, the common difference, \(d = 17.5/5 = 3.5\)
Hence, the series is: 5, 8.5, 12, 15.5, 19, 22.5

6. The first, tenth and last terms of an arithmetic progression are 9, 40.5 and 425.5, respectively. Find (a) the number of terms, (b) the sum of all the terms, and (c) the 70th term.

(a) \( a = 9 \) and the 10th term is: \( a + (10 - 1)d = 40.5 \)

i.e. \( 9 + 9d = 40.5 \) and \( 9d = 40.5 - 9 = 31.5 \)

hence \( d = \frac{31.5}{9} = 3.5 \)

Last term is given by: \( a + (n - 1)d \)

i.e. \( 9 + (n - 1)(3.5) = 425.5 \)

i.e. \( (n - 1)(3.5) = 425.5 - 9 = 416.5 \)

and \( n - 1 = \frac{416.5}{3.5} = 119 \)

Hence, the number of terms, \( n = 120 \)

(b) Sum of all the terms, \( S_n = \frac{n}{2}[2a+(n-1)d] = \frac{120}{2}[2(9)+(120-1)(3.5)]=60[18+416.5] \)

\( = 26070 \)

(c) The 70th term is: \( a + (n - 1)d = 9 + (70 - 1)(3.5) = 9 + 69(3.5) = 250.5 \)

7. On commencing employment a man is paid a salary of £16 000 per annum and receives annual increments of £480. Determine his salary in the 9th year and calculate the total he will have received in the first 12 years.

The series is: 16 000, 16 480, 16 960, ... to 9 terms, i.e. \( a = 16 000, \ d = 480 \) and \( n = 9 \)

Salary after 9 years = \( a + (n - 1)d = 16 000 + (9 - 1)(480) = 16 000 + 3840 = £19 840 \)

Thus, total salary in 12 years,

\[ S_n = \frac{n}{2}[2a+(n-1)d] = \frac{12}{2}[2(16000)+(12-1)(480)] = 6[32 000 + 5280] \]

\( = 6(37 280) = £223 680 \)
8. An oil company bores a hole 80 m deep. Estimate the cost of boring if the cost is £30 for drilling the first metre with an increase in cost of £2 per metre for each succeeding metre.

The series is: 30, 32, 34, … to 80 terms, i.e. \( a = 30, \ d = 2 \) and \( n = 80 \)

Thus, total cost,

\[
S_n = \frac{n}{2} \left[ 2a + (n-1)d \right] = \frac{80}{2} \left[ 2(30) + (80-1)(2) \right] = 40 \left[ 60 + 158 \right] = 40(218) = £8720
\]
EXERCISE 81 Page 172

1. Find the 10th term of the series 5, 10, 20, 40, ...

The 10th term of the series 5, 10, 20, 40, … is given by:

\[ a \cdot r^{n-1} \]

where \( a = 5 \), \( r = 2 \) and \( n = 10 \)

i.e. the 10th term = \( a \cdot r^{n-1} = (5)(2)^{10-1} = (5)(2)^9 = 5(512) = 2560 \)

2. Determine the sum of the first 7 terms of the series 0.25, 0.75, 2.25, 6.75, ...

0.25, 0.75, 2.25, 6.75, … is a GP with a common ratio \( r = 3 \)

The sum of \( n \) terms, \( S_n = \frac{a(r^n - 1)}{(r-1)} \)

Hence, the sum of the first 7 terms, \( S_7 = \frac{0.25(3^7 - 1)}{(3-1)} = \frac{0.25(2187 - 1)}{2} = \frac{0.25(2186)}{2} = 273.25 \)

3. The 1st term of a geometric progression is 4 and the 6th term is 128. Determine the 8th and 11th terms.

The 6th term is given by: \( a \cdot r^5 = 128 \) i.e. \( 4r^5 = 128 \) and \( r^5 = \frac{128}{4} = 32 \)

Thus, \( r = \sqrt[5]{32} = 2 \)

Hence, the 8th term is: \( a \cdot r^7 = 4(2)^7 = 4(128) = 512 \)

and the 11th term is: \( 4(2)^{10} = 4(1024) = 4096 \)

4. Find the sum of the first 7 terms of the series 2, 5, 12.5, ... (correct to 4 significant figures).

Common ratio, \( r = \frac{ar}{a} = \frac{5}{2} = 2.5 \) (also, \( \frac{ar^2}{ar} = \frac{12.5}{5} = 2.5 \))

Sum of 7 terms, \( S_n = \frac{a(r^n - 1)}{(r-1)} = \frac{2(2.5^7 - 1)}{(2.5-1)} = \frac{2(610.35 - 1)}{1.5} = 812.5 \), correct to 4 significant figures
5. Determine the sum to infinity of the series 4, 2, 1, ...

The series is a GP where \( r = \frac{2}{4} = 0.5 \) and \( a = 4 \)

Hence, sum to infinity, \( S_\infty = \frac{a}{1-r} = \frac{4}{1-(0.5)} = \frac{4}{0.5} = 8 \)

6. Find the sum to infinity of the series \( 2 \frac{1}{2}, -1 \frac{1}{4}, -\frac{5}{8}, ... \)

The series is a GP where \( r = -\frac{1.25}{2.5} = -0.5 \) and \( a = 2.5 \)

Hence, sum to infinity, \( S_\infty = \frac{a}{1-r} = \frac{2.5}{1-(-0.5)} = \frac{2.5}{1.5} = \frac{5}{3} = 1 \frac{2}{3} \)
1. In a geometric progression the 5th term is 9 times the 3rd term and the sum of the 6th and 7th terms is 1944. Determine (a) the common ratio, (b) the first term and (c) the sum of the 4th to 10th terms inclusive.

(a) The 5th term of a geometric progression is: \(ar^4\) and the 3rd term is: \(ar^2\)

Hence, \(ar^4 = 9ar^2\) from which, \(\frac{r^4}{r^2} = 9\) i.e. \(r^2 = 9\)

from which, the common ratio, \(r = 3\)

(b) The 6th term is \(ar^5\) and the 7th term is \(ar^6\)

Hence, \(ar^5 + ar^6 = 1944\)

Since \(r = 3\), \(243a + 729a = 1944\)

i.e. \(972a = 1944\) and first term, \(a = \frac{1944}{972} = 2\)

(c) Sum of the 4th to 10th terms inclusive is given by:

\[
S_{10} - S_3 = \frac{a(r^{10} - 1)}{(r-1)} - \frac{a(r^3 - 1)}{(r-1)} = \frac{2(3^{10} - 1)}{3 - 1} - \frac{2(3^3 - 1)}{3 - 1}
\]

\[
= (3^{10} - 1) - (3^3 - 1) = 3^{10} - 3^3 = 59049 - 27 = 59022
\]

2. Which term of the series 3, 9, 27, ... is 59049?

First term, \(a = 3\), common ratio, \(r = 3\) and the \(n\)th term of a geometric series is: \(ar^{n-1}\)

Thus, \(59049 = 3(3)^{n-1}\)

from which, \(3^{n-1} = \frac{59049}{3} = 19683\)

Taking logarithms gives: \(\lg 3^{n-1} = \lg 19683\)

i.e. \((n - 1)\lg 3 = \lg 19683\) and \(n - 1 = \frac{\lg 19683}{\lg 3} = 9\)

from which, \(n = 9 + 1 = 10\) i.e. 59049 is the 10th term of the series 3, 9, 27, ...
3. The value of a lathe originally valued at £3000 depreciates 15% per annum. Calculate its value after four years. The machine is sold when its value is less than £550. After how many years is the lathe sold?

\[ a = 3000, \ r = 0.85 \text{ and } n = 4, \]

hence the value after one year is: \((0.85)(3000)\)

the value after two years is: \((0.85)^2(3000)\)

and the value after four years is: \((0.85)^4(3000) = £1566\)

Let \(550 = (0.85)^n(3000)\) from which, \(\frac{550}{3000} = 0.85^n\)

Taking logarithms to base 10 of both sides of the equation gives:

\[
\lg \left( \frac{550}{3000} \right) = n \lg 0.85
\]

Hence, number of years before lathe is sold, \(n = \frac{\lg \left( \frac{550}{3000} \right)}{\lg 0.85}\]

\[= 10.44 = 11 \text{ years to the nearest whole number}\]

4. If the population of Great Britain is 55 million and is decreasing at 2.4% per annum, what will be the population in five years’ time?

GB population now = 55 million, population after one year = \(0.976 \times 55\) million

Population after two years = \((0.976)^2 \times 55\) million

Hence, population after five years = \((0.976)^5 \times 55 = 48.71\) million

5. 100 g of a radioactive substance disintegrates at a rate of 3% per annum. How much of the substance is left after 11 years?

\[ a = 100, \ r = 0.97 \text{ and } n = 11 \]

and the amount of substance left after 11 years is: \((0.97)^{11}(100) = 71.53\) g
6. If £250 is invested at compound interest of 6% per annum, determine (a) the value after 15 years, (b) the time, correct to the nearest year, it takes to reach £750

(a) First term, \( a = £250 \), common ratio, \( r = 1.06 \)

Hence, value after 15 years = \( ar^{15} = (250)(1.06)^{15} = £599.14 \)

(b) When £750 is reached, \( 750 = ar^n \)

i.e. \( 750 = 250(1.06)^n \)

and \( \frac{750}{250} = 1.06^n \) i.e. \( 3 = 1.06^n \)

Taking logarithms gives: \( \lg 3 = \lg (1.06)^n = n \lg 1.06 \)

from which, \( n = \frac{\lg 3}{\lg 1.06} = 18.85 \)

Hence, it will take 19 years to reach more than £750

7. A drilling machine is to have eight speeds ranging from 100 rev/min to 1000 rev/min. If the speeds form a geometric progression, determine their values, each correct to the nearest whole number.

First term, \( a = 100 \) rev/min

The 8th term is given by: \( ar^{8-1} = 1000 \) from which, \( r^7 = \frac{1000}{100} = 10 \) and \( r = \sqrt[7]{10} = 1.3895 \)

Hence, 1st term is 100 rev/min

2nd term is \( ar = (100)(1.3895) = 138.95 \)

3rd term is \( ar^2 = (100)(1.3895)^2 = 193.07 \)

4th term is \( ar^3 = (100)(1.3895)^3 = 268.27 \)

5th term is \( ar^4 = (100)(1.3895)^4 = 372.76 \)

6th term is \( ar^5 = (100)(1.3895)^5 = 517.96 \)

7th term is \( ar^6 = (100)(1.3895)^6 = 719.70 \)
8th term is \( ar^7 = (100)(1.3895)^7 = 1000 \)

Hence, correct to the nearest whole numbers, the eight speeds are:

100, 139, 193, 268, 373, 518, 720 and 1000 rev/min
1. Evaluate: (a) \( ^9C_6 \)  (b) \( ^3C_1 \)

(a) \( ^9C_6 = \frac{9!}{6!(9-6)!} = \frac{9!}{6!3!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{(6 \times 5 \times 4 \times 3 \times 2)(3 \times 2)} = \frac{9 \times 8 \times 7}{3 \times 2} = 3 \times 4 \times 7 = 84 \)

(b) \( ^3C_1 = \frac{3!}{1!(3-1)!} = \frac{3!}{1!2!} = \frac{3 \times 2}{(1)(2)} = 3 \)

2. Evaluate: (a) \( ^6C_2 \)  (b) \( ^8C_3 \)

(a) \( ^6C_2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2!4!} = \frac{6 \times 5 \times 4 \times 3 \times 2}{2(4 \times 3 \times 2)} = \frac{6 \times 5}{2} = 3 \times 5 = 15 \)

(b) \( ^8C_3 = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{(5 \times 4 \times 3 \times 2)(3 \times 2)} = \frac{8 \times 7 \times 6}{3 \times 2} = 4 \times 7 \times 2 = 56 \)

3. Evaluate: (a) \( ^4P_2 \)  (b) \( ^7P_2 \)

(a) \( ^4P_2 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4 \times 3 \times 2}{(2)} = 4 \times 3 = 12 \)

(b) \( ^7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2}{(3 \times 2)} = 7 \times 6 \times 5 \times 4 = 840 \)

4. Evaluate: (a) \( ^10P_3 \)  (b) \( ^8P_3 \)

(a) \( ^10P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{7 \times 6 \times 5 \times 4 \times 3 \times 2} = 10 \times 9 \times 8 = 720 \)

(b) \( ^8P_3 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{(3 \times 2)} = 8 \times 7 \times 6 \times 5 \times 4 = 6720 \)