Answers To Chapter 9

**Review Questions**

1. **Answer d.** Other benefits include a more stable employment situation, more interesting and challenging work, and access to occupations with more prestige and more desirable working conditions.

2. **Answer d.** The costs associated with education include direct expenses, forgone earnings, and psychic losses.

3. **Answer d.** While discounting will also adjust for differences in purchasing power if the discount rate includes a premium for expected inflation, it is also possible to adjust for inflation using a price index for different years. Even if all the benefits are expressed in real terms, it would still be necessary to discount future benefits.

4. **Answer a.**

   \[
   \frac{1,000}{(1 + 0.06)^{10}} = 558.39.
   \]

5. **Answer a.** Substituting \(B = 10,000\), \(r = 0.06\), and \(T = 40\) into the annuity formula yields

   \[
   PVB = \frac{1 - (1 + 0.06)^{-40}}{0.06} \Rightarrow PVB = 150,462.97
   \]

6. **Answer b.** Setting the present value of the benefits equal to the costs

   \[
   \Rightarrow \frac{2,500}{(1 + r)^2} = \frac{2,000}{(1 + r)} \Rightarrow (1 + r)^2 = \frac{2,500}{2,000}
   \]

   \[
   \Rightarrow 1 + r = \sqrt{1.25} \Rightarrow r = 0.118 \Rightarrow r = 11.8\%.
   \]

7. **Answer c.** An increase in the retirement age effectively increases \(T\), the length of time over which the benefits to education can accrue, raising the present value of the benefit stream. On the other hand, an increase in \(r\) would lower the present value of the benefit stream. The higher wages and decreased aptitude associated with high school graduates will tend to raise the costs of additional educational investments by increasing the forgone earnings and raising the psychic costs of education.

8. **Answer b.** Even if the value of \(B\) were falling for most individuals, an increase in \(T\) could more than compensate, thus raising the present value of the benefit stream.
9. **Answer a.** The net gain from education is greatest at 12 years. The net gain, represented by the distance between points \( a \) and \( b \), equals $1,000,000 and is greater than that attained at any other education level. For example, at 14.5 years, the net gain is zero, while at 16 years the net gain is only $200,000 (the distance between points \( e \) and \( d \)). At 20 years of education, the costs actually exceed the benefits.

10. **Answer c.** At 16 years, the net gain is $1,600,000 (the difference between points \( e \) and \( c \)), while at 12 years the net gain is only $1,000,000. At 14.5 the net gain is something less than $1,000,000 because additional costs have been incurred, but no additional benefits. Likewise, at 20 years, there are additional costs but no added benefits, leaving net benefits less than $1,600,000.

11. **Answer c.** Since only the higher-productivity individuals (the type B individuals) will have an incentive to attain 16 years of education, this level of achievement will be an effective way to distinguish between types of workers. The reason the low-productivity individuals voluntarily choose the lower education level is because of the higher costs they face for attaining a given level of education.

12. **Answer b.** The socially optimal level of education to use as a screening device would be that level just slightly past point \( f \) (14.5 years). This is the lowest threshold that firms could set and still have type A workers choose less than the threshold amount and type B workers above the threshold amount. In a world where education’s only value is as a signal, it does not make sense from the point of view of society as a whole to devote any resources to education beyond that level needed to maintain the signaling function.

13. **Answer d.** Answers b and c account for the steepening of the age-earnings profile, while Answer a accounts for the flattening of the profile as age increases.

14. **Answer b.** Age-earnings profiles steepen when an individual receives more on-the-job training. On-the-job training, in turn, is more likely to be acquired by those for whom the costs of training are lower, holding all else constant. Those who have completed higher levels of schooling have shown that they can learn more quickly, which implies that the cost of educational investments will be lower for these individuals.

15. **Answer d.** Steepening of the age-earnings profile is usually attributable to on-the-job training. Such training becomes a better investment for women as their worklives increase (\( T \) increases).

16. **Answer b.** The higher wages attained by those completing higher levels of education are, for the most part, the benefits to investing in human capital. However, these benefits must be weighed against the costs. For the investment to be worthwhile, not only must the internal rate of return be positive, but it must also exceed the person’s discount rate.

17. **Answer c.** High-ability people typically earn more than lower-ability people, holding all else constant. Since education and ability are positively correlated, measures of the rate of return to education that ignore ability may attribute all of the earnings gap to education itself, when in fact some of the difference is due to the higher ability of the more educated person.

18. **Answer d.** For those who actually made the investment, the return may be understated since they probably would have been a worse than average worker in the unskilled profession. On the other hand, for those who did not make the investment, the potential return is overstated since they may be a worse than average worker in jobs requiring greater skill.
19. **Answer a.** If a certain level of education already has signaling value, higher levels of education may actually destroy the signaling value of education. **Answer b,** far from being a sign of wasted resources, represents a situation where the level of education has reached its optimal social level. **Answer c** is a sign that the investment is acceptable since the investor will earn just a normal rate of return.

20. **Answer b.** The initial shortage is the difference between the quantity demanded off the new demand curve (28) and the quantity supplied (18).

   For readers wishing to derive these answers more precisely, note that the equations underlying Questions 20–24 are
   \[
   \text{Demand 1: } L_D = 30 - W, \\
   \text{Demand 2: } L_D = 40 - W, \\
   \text{Supply: } L_S = 1.5W.
   \]

21. **Answer b.** With the quantity of labor supplied fixed at 18, the wage must rise to $22 to clear the market.

22. **Answer d.** When the labor supply plans of all workers have been realized, the surplus will be the difference between the quantity supplied and quantity demanded at the wage of $22.

23. **Answer a.** With the quantity of labor supplied now at 33, the wage must fall to $7 to clear the market.

24. **Answer d.** The magnitude of the slope of the demand curves (as it appears on the graph) is one, while the slope of the supply curve is two-thirds. If workers had rational expectations there would be no boom-and-bust cycle. If workers were adapting their expectations based on the past behavior of wages, the cycle would tend to diminish.

25. **Answer b.** Indifference curves represent the education and wage combinations that keep worker utility constant at a particular level, not what firms have to do to attract workers. In this case, since higher wages accompany higher levels of education, the implicit message is that higher levels of education are perceived by workers as a **bad** since they are costly. Therefore, in order to keep workers at the same level of utility the wage, a **good,** must rise. Note that with this kind of preference structure, workers prefer movements toward combinations that involve a higher wage and less education.

26. **Answer d.** An isoprofit curve is a constraint from the firm’s point of view. It represents what the firm is willing and able to pay workers with different levels of education, not what workers demand. If the firm perceives that workers with higher levels of education are more productive, either because education makes them more productive or because it identifies workers who are inherently more productive, it should be willing and able to pay them a higher wage and still stay at the zero (normal) profit level.
27. **Answer d.** The steeper the indifference curve, the more averse the person is to additional years of education. On the other hand, the steeper the isoprofit curve, the more the firm is willing to pay for additional years of education. The matching of person B with firm Z suggests that the person least averse to additional schooling is matched with the firm most willing to pay for additional education. Since each individual is free to choose from the combinations offered by both firms Y and Z, the implicit assumption must be that A and B are mobile enough to be able to accept either employment opportunity. Such a matching process also typically assumes that workers are accurately informed about the wage and education combinations offered by the different firms.

### Problems

28. Substituting $B = 3,000$, $r = 0.06$, and $T = 10$ into the annuity formula yields a total present value of benefits equal to

$$PVB = \frac{1 - \frac{1}{(1 + 0.06)^{10}}}{0.06} \Rightarrow PVB = 22,080.26$$

The total cost of the investment is the $2,000 for tuition and books plus $20,000 in lost earnings for a total cost of $22,000. The costs need not be discounted since they all occur in the current year (year 0). Since the present value of the benefits exceeds the present value of the costs, this is a good investment.

*29a. Setting the present value of the benefits equal to the present value of the cost yields

$$\Rightarrow \frac{2,000}{1 + r} + \frac{2,000}{(1 + r)^2} = 3,500$$

$$\Rightarrow \frac{2,000 + 2,000r + 2000}{(1 + r)^2} = 3,500$$

$$\Rightarrow 4,000 + 2,000r = 3,500(1 + r)^2$$

$$\Rightarrow 4,000 + 2,000r = 3,500(1 + 2r + r^2)$$

$$\Rightarrow 3,500r^2 + 5,000r - 500 = 0$$

$$\Rightarrow 35r^2 + 50r - 5 = 0.$$  

Applying the quadratic formula yields

$$r = \frac{-50 \pm \sqrt{50^2 - 4(35)(-5)}}{2(35)} \Rightarrow r = 0.0938 \text{ or } 9.38\%.$$  

29b. Since the internal rate of return exceeds the interest rate on alternative investments, the training program is a good investment for the worker.
29c. Any discount rate that is less than or equal to the internal rate of return would still make this a worthwhile investment. A person may have a higher discount rate than the market interest rate if he or she is very present-oriented. For example, a person may discount future benefits at a very high rate because of a fear that he or she may not live long enough to enjoy the benefits.

29d. Since older workers have a shorter time period over which to reap the benefits of the investment, the present value of the entire benefit stream will be lower, holding all else constant. Also, since older workers typically receive a higher wage than younger workers, the earnings that they forgo during training are higher, raising the cost of the investment. The tendency of on-the-job training to diminish with age leads to a flattening out of the age-earnings profile, thus contributing to its concave shape.

30a. Since the costs occur in the current period they are not discounted. The present value of the benefits equals

\[
\text{PVB} = \frac{\$6,000}{1+0.06} + \frac{\$2,000}{(1+0.06)^2} = $11,000.40.
\]

Therefore, the present value of the benefits exceeds the present value of the costs by $1,000.40.

30b. Substituting \( r = 0.06 \) and \( p = 0.04 \) into the formula for the real interest rate \( (i) \) yields

\[
i = \frac{0.06 - 0.04}{1 + 0.04} = 0.01923 \quad \text{or} \quad 1.923\%.
\]

30c. To convert nominal values to real values, divide the nominal value by the price index and multiply the result by 100. This yields

- Real cost in year 0 = 10,000,
- Real benefit in year 1 = 5,769.23,
- Real benefit in year 2 = 5,547.34.

30d. Since the costs occur in the current period they are not discounted. The present value of the real benefits equals

\[
\text{PVB} = \frac{5,769.23}{1.01923} + \frac{5,547.34}{(1.01923)^2} = $11,000.40.
\]

Therefore, the present value of the benefits exceeds the present value of the costs by 1,000.40 in real terms.

30e. Comparing the Answers to 30a and 30d suggests that discounting nominal values by the market interest rate is the same as converting nominal values to real and then discounting by the real interest rate. Because the market interest rate adjusts to reflect anticipated inflation, it is not necessary to deflate future benefits by the anticipated price indices.

31a. For those who have actually made the educational investment, selection bias leads to an understatement of the true return, which suggests that the upper end of the range of estimates is more appropriate. On the other hand, for those who have not made the investment, selection bias tends to overstate the rate of return that is possible on educational investments. This suggests that the lower end of the range of estimates would be more appropriate.
31b. Selection bias and ability bias are both examples of the omitted variable problem discussed in the appendix to Chapter 1. Selection bias is a problem that stems from a failure to account for the comparative advantage people have in different occupations. For example, someone who successfully completes a college education probably has a set of interests and abilities that would not be well utilized in a work environment suitable for the typical high school graduate. That is presumably part of the reason the person chose to attend college in the first place. Therefore, simply comparing the average earnings of high school and college graduates understates the benefits to the person choosing to attend college. Such a simple comparison does not account for the fact that the person would probably not have been as successful as the typical high school graduate had he or she not attended college. If one could control for the different aptitudes and interests of each individual, there would be no selection bias. The problem is that these differences are very difficult to observe and measure and so in some studies their effects go unaccounted for. In the same way, ability bias is simply a failure to control for the differences in ability that occur between those achieving different levels of education. Since ability is difficult to define and measure, it often goes unaccounted for. To do so, however, overstates the return to education provided individuals attaining higher levels of education also have more general ability. The increase in wages earned by the more educated and more able workers is all attributed to education if measures of ability are not also included in the analysis.

32a. For a type A person, the net benefit from 12 years of education is $1,000,000 (distance ab), while the net benefit from 16 years of education is $800,000 (distance ed). The optimal level of education is 12 years.

32b. For a type B person, the net benefit from 12 years of education is $1,000,000 (distance ab), while the net benefit from 16 years of education is $1,200,000 (distance ec). The optimal level of education is 16 years.

32c. Since only type B workers have an incentive to acquire 16 years of education, that level of education could be used to distinguish high productivity workers from low-productivity workers.

32d. Figure 9-10 shows the new wage schedules given the decrease in the hiring standard.

![Figure 9-10](image-url)
Note that type A individuals will now have an incentive to attain 14 years of education since the net gain of $1,400,000 (distance $ed$) exceeds the $1,000,000 return on 12 years of education (distance $ab$). Type B individuals will also have an incentive to attain 14 years of education since the net gain of $1,600,000 (distance $ec$) exceeds the $1,000,000 return on 12 years of education. Since both types of workers would have an incentive to acquire 14 years of education, that threshold would be an ineffective signal of worker productivity.

32e. Figure 9-11 shows the new wage schedules given the increase in the hiring standard. Note that type A individuals will now have an incentive to attain only 12 years of education since the net gain of $1,000,000 (distance $ab$) exceeds the $200,000 return on 18 years of education (distance $ed$). Type B individuals will also have an incentive to attain just 12 years of education since the net gain of $1,000,000 exceeds the $800,000 return on 18 years of education (distance $ec$). Since both types of worker would have an incentive to acquire just 12 years of education, that threshold would be an ineffective signal of worker productivity.

![Figure 9-11](image1)

33a. Equilibrium $\Rightarrow L_D = L_S$

$$10 - 0.5W = 0.5W$$

$$W^* = $10 \Rightarrow L^* = 5.$$ 

The initial equilibrium is represented by point $a$ in Figure 9-12.

![Figure 9-12](image2)
33b. Equilibrium $\Rightarrow L_D = L_S$

$\Rightarrow 12 - 0.5W = 0.5W \Rightarrow W^* = 12 \Rightarrow L^* = 6.$

The new equilibrium is represented by point e in Figure 9-12.

33c. With the quantity of labor supplied fixed temporarily at 5, the market clears where

$12 - 0.5W = 5 \Rightarrow W^* = 14.$

This outcome is represented by point b in Figure 9-12.

33d. With the wage rising to $14, an incentive will be created for the quantity of labor supplied to increase to

$L_S = 0.5W = 0.5(14) \Rightarrow L_S = 7.$

This outcome is represented by point c in Figure 9-12.

33e. With the quantity of labor supplied fixed temporarily at 7, the market will clear at

$12 - 0.5W = 7 \Rightarrow W^* = 10.$

This outcome is represented by point d in Figure 9-12.

33f. With the wage at $10, there will be an incentive to supply labor to the point where

$L_S = 0.5W = 0.5(10) \Rightarrow L_S = 5.$

This outcome is represented by point a in Figure 9-12.

33g. The market is not moving closer to the market-clearing values associated with the demand curve $D_2$. Rather, the market continues to cycle around the true equilibrium value.

33h. The cycle will not converge to the true equilibrium since the magnitude of the slope of the demand curve (as it appears on the graph) is the same as the slope of the supply curve. For this model to converge, the demand curve must be flatter than the supply curve.

34a. Provided person B were willing and able to switch to employer Z (i.e., move from point a to point b), the 4-year increase in education would result in a wage increase of slightly over $4.

34b. Given the initial indifference curve $B_1$, the person would require an increase in the wage of just over $2 to undertake the costs of an additional four years of education. This difference of just over $2 represent the difference in the wage between points a and c.

34c. The additional years of education would be a good investment for person B since the benefits exceed the minimum necessary to keep utility constant. The result is an increase in utility as B moves from point a to point c, and from indifference curve $B_1$ to indifference curve $B_2$. On the other hand, person A is already at his or her optimum wage and education combination. Any increases in education beyond point a will result in a lower level of utility since the increases will result in a smaller increase in the wage than is necessary to compensate person A for the increased costs.
Applications

35. The simplest explanation is that enrollment rates were rising, despite the cost increases, because the benefits of a college education were also rising. (According to the article, male college graduates earned on average 23.8% more than high school graduates in 1979, but by 1986 the difference had risen to 39.2%. For women, the difference rose from 27.9% to 40.5% over the same period.) Human capital investment decisions are made by comparing costs and benefits.

36. This explanation is very plausible and highlights the uncertain nature of the benefits associated with educational investments. Decisions about particular educational investments are based on expectations of future earnings, formed largely through contacts with friends, family, neighborhood acquaintances, and other role models. It seems very plausible, given the small percentage of women with graduate degrees in business in the early 1970s, that information on future career and earnings prospects would have been relatively hard to come by. Recent experiences suggest that the initial estimates made by many women of the benefits of an M.B.A. degree may have been overly optimistic. As this information has circulated to younger women, it apparently has had the dampening effect on enrollment rates that the human capital investment framework would predict.

*37a. The parameter $r$ could be estimated using least squares regression analysis. Letting $S$ be the independent variable, $\ln Y_S$ the dependent variable, and $\ln Y_0$ the constant (or intercept) term, an estimate of $r$ could be derived by fitting a least squares regression line to cross-section data on schooling and earnings.

*37b. A large number of variables that can influence earnings have been left out of this model. For example, the amount of ability, on-the-job training, experience, and employee benefits a person receives all influence earnings. However, these omitted variables only bias the estimate of $r$ if they are correlated with the level of education. For example, some measure of individual ability must be included in the regression (making this a multiple regression analysis). Since education and ability are most likely positively correlated, failure to include a measure of ability would bias the estimate of $r$ upward.

*37c. Unless the regression included a good measure of individual interests and aptitudes, the estimate would suffer from selection bias. The estimate of $r$ would overstate the rate of return that could be expected by someone who has not invested in additional years of schooling, and would understate the return attained by those who already have invested in additional years of schooling.

*37d. The rate of return would be 8.4%.

\[
 r = \frac{\ln 35,000 - \ln 25,000}{4} = 0.084
\]

*37e. Education would be a good investment provided this rate of return exceeded the rate of return available on other comparable investments.

38a. For a type A person, the net benefit from 12 years of education is $1,000,000 (distance $ab$), while the net benefit from 16 years of education is $1,400,000 (distance $ed$). The optimal level of education is 16 years.

38b. For a type B person, the net benefit from 12 years of education is $1,000,000 (distance $ab$), while the net benefit from 16 years of education is $1,600,000 (distance $ec$). The optimal level of education is 16 years.
38c. Since both types of workers have an incentive to acquire 16 years of education, that level of education could not be used to distinguish high-productivity workers from low-productivity workers. Lowering the cost of education has increased the incentive to invest in education, but in so doing it has taken away education’s signaling value, thus eliminating the social benefit society derives from education in this model.

38d. Grade inflation may lower the cost of education perceived by individuals, particularly the psychic costs, by reducing the difficulty and anxiety associated with schooling. As shown in the previous questions, however, lowering the cost of education can destroy the ability of education to function as an effective signal of the most productive individuals.

39a. The law would force person A to move from point $a$ to point $b$. This would force person A to a lower indifference curve. Person B would not be affected.

39b. If person A is restricted to employer Y, the law would force person A to move from point $a$ to point $c$. This would lead to a greater reduction in utility than in the previous problem. Person B would not be affected.